Mu Alpha Theta (M A TH)

NATIONAL HIGH SCHOOL AND JUNIOR COLLEGE MATHEMATICS CLUB

601 Elm Avenue, Room 423, Norman, Oklahoma 73019

405-325-4489

Cosponsored by The Mathematical Association of America and The National Council of Teachers of Mathematics

April 24, 1989

[redacted]

Dear Tammy:

As national secretary-treasurer for Mu Alpha Theta, it is my pleasure to inform you that you have been selected as the 1989 winner of the Andree Award, offered by Mu Alpha Theta in honor of Dr. Richard Andree and Mrs. Josephine Andree, who founded Mu Alpha Theta some thirty years ago.

As you know this award is given to an outstanding student who is planning a career in the teaching of mathematics and whose college major reflects this interest. The cash award is $1000, and a check for that amount is enclosed.

We hope that you will be able to attend the annual convention of Mu Alpha Theta to be held in Tampa, Florida, August 3-8, 1989 at the Hyatt Regency Westshore, at which time your award will be formally announced and you will be presented with a commemorative plaque. If you are able to attend, Mu Alpha Theta will pay for your registration, meals and lodging. If you will be able to attend, please let us know.

You are to be congratulated for having won this award, for the other nominees were of outstanding quality. We hope this honor will serve to strenghten your commitment to a career in mathematics teaching.

Sincerely,

Thomas J. Hill

Thomas J. Hill
Secretary-Treasurer

TJH/dr
cc: Mrs. Lois McMullan
East Central Community College
Josephine Andree
Governing Council

EAST CENTRAL COMMUNITY COLLEGE

DECATUR, MISSISSIPPI 39327

OFFICE OF THE PRESIDENT *601-635-2111*

May 5, 1989

[redacted]

Dear Tammy:

On behalf of the Board of Trustees, administration, faculty, and staff of East Central Community College I want to congratulate you for having been awarded the Mu Alpha Theta Andree Award for 1989. To have been named the first recipient of this prestigious national award is a signal honor. We at East Central Community College are extremely proud of you.

You have brought much honor to East Central during the two years you have been here as a student. Also, you have set an example of excellence in learning that will, I'm certain, have lasting positive effects on others for many years to come.

Best wishes for all of your future endeavors. It will be exciting to observe your future unfold.

With warmest regards, I am

Sincerely yours,

Eddie M. Smith

Eddie M. Smith
President

EMS:las

cc: Governor Ray Mabus
Board of Trustees
Mrs. Lois McMullan
Dr. Shelby Harris
Mr. Mac McClellan

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The Mathematical Log

VOLUME 33, NUMBER 3 OCTOBER 1989

COMPETITIONS CAN BE FUN!

School Contest Marks 40th Anniversary

The Mathematical Association of America recently noted the fortieth anniversary of the American High School Mathematics Examination, currently written by almost 400 000 students in over 6000 high schools in the U.S., Canada, and abroad. Designed to stimulate interest in mathematics at high school level, the contest--now in thirty-question, multiple-choice format--was cosponsored by the MAA and the Society of Actuaries. Mu Alpha Theta became a sponsor in 1965, NCTM in 1968, and other groups followed.

Of the approximately 10 000 000 students who participated in the first thirty-nine contests, only 28 have written perfect papers, the MAA report notes.

Focus, the MAA newsletter, invited its readers to try five sample questions from recent examinations. "Remember that no calculators are permitted!" the report directs.

1. Which of the following is closest to $\sqrt{65} - \sqrt{63}$?
   (A) 0.12 (B) 0.13 (C) 0.14 (D) 0.15
   (E) 0.16

2. A ball was floating in a lake when the lake froze. The ball was removed (without breaking the ice), leaving a hole 24 cm across at the top and 8 cm deep. What was the radius of the ball (in centimeters)?
   (A) 8 (B) 12 (C) 13 (D) $8\sqrt{3}$ (E) $6\sqrt{6}$

3. Each integer 1 through 9 is written on a separate slip of paper and all nine slips are put into a hat. Jack picks one of these slips at random and puts it back. Then Jill picks a slip at random. Which digit is most likely to be the units digit of the sum of Jack's integer and Jill's integer?
   (A) 0 (B) 1 (C) 8 (D) 9 (E) Each is equally likely.

4. If $\sin(x) = 3\cos(x)$ then what is $\sin(x)\cos(x)$?
   (A) $\frac{1}{6}$ (B) $\frac{1}{5}$ (C) $\frac{2}{9}$ (D) $\frac{1}{4}$ (E) $\frac{3}{10}$

5. Suppose that p and q are positive numbers for which $\log_9(p) = \log_{12}(q) = \log_{16}(p + q)$
   What is the value of $\frac{q}{p}$?
   (A) $\frac{4}{3}$ (B) $\frac{1}{2}(1 + \sqrt{3})$ (C) $\frac{8}{5}$ (D) $\frac{1}{2}(1 + \sqrt{5})$
   (E) $\frac{16}{9}$

Problems, answers, and solutions to all AHSME papers are available in a group of New Mathematics Library publications (to date, vols. 5, 17, 25, and 29).

Answers to the above questions appear elsewhere in this Log issue.

Group 'Power Question' Unique Challenge

Representing a high level contest challenge for a student group at the level of Mu Alpha Theta seniors, the annual ARML Power Question is--reliably--one of the best to come across the Mathematical Log editorial desk. The 1989 question has just arrived from Harry D. Ruderman, an outstanding friend and supporter of Mu Alpha Theta. Dr. Ruderman notes that question preparation is teamwork with colleagues Gil Kessler and Larry Zimmerman, but admits to being "quite proud of the Pythagorean polygon."

The 1989 Power Question follows, with its sample solution, also supplied by Dr. Ruderman, to appear in our Tall Timbers supplement.

Care to get together with mathematically inclined friends and give the Power Question a cooperative try?

* * *

A convex n-gon will be called "Pythagorean" if it has integer sides, it is cyclic, and its longest side is a diameter for its circumscribing circle. It shall be denoted by Pn or Pn:(a,b,...), where a,b,... are the lengths of its sides. We shall always use the letter d for its longest side. [Thus P3 is a Pythagorean triangle. Note that it would be a right triangle.]

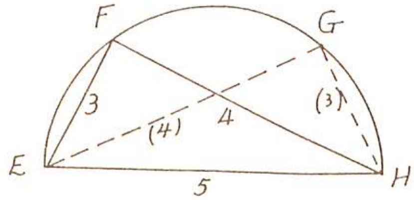
I.

[There is a theorem which states (in part) that: if a prime d is the hypotenuse of a Pythagorean triangle, then d^2 is the hypotenuse of two Pythagorean triangles, d^3 is the hypotenuse of three Pythagorean triangles, etc.]

A. Find two P3's for which d = 25.
B. Find three P3's for which d = 125.

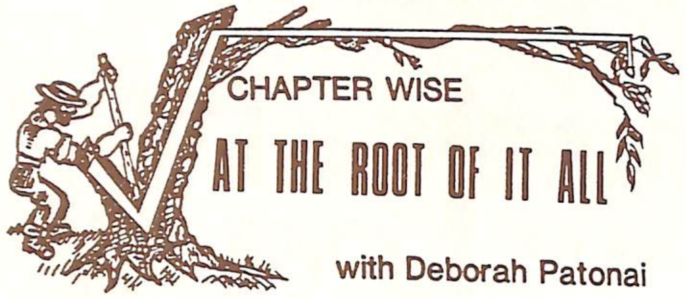
II.

Ptolemy's Theorem says: A convex quadrilateral is cyclic if and only if the product of its diagonals equals the sum of the products of the two pairs of opposite sides.



A. If the P3:(3,4,5) is reflected as above, a quadrilateral EFGH can be formed (it will not be a P4, as FG is not an integer). Multiplying each side by 5 produces a

(See "Group 'Power Question,'" page four)



with Deborah Patonai

"Once Mu Alpha Theta, always Mu Alpha Theta," as long-time national secretary-treasurer Harold Huneke has told more than one convention audience, in encouraging students and sponsors to take continuing pride in their affiliation, and, over the years, to keep in touch. When your Editor flew East from Seattle convention in 1987, his seatmate was an engineer, now in management, who had been Mu Alpha Theta in the late fifties ... and remained rightly proud of it. Here, Activities Editor Deborah Patonai tells of a unique effort to maintain a bond which now extends to hundreds of thousands of young Americans.

Ms. Patonai writes:

Have you ever had a "neat" idea ... but, for some reason or other, you've never developed or pursued it? Well, as a Mu Alpha Theta advisor who has attended the last 12 national conventions, I've always thought that inviting all my former Mu Alpha Theta conventioneers together for a "convention reunion" would be interesting and fun. One day I may be able to put my idea into action. However, a Mu Alpha Theta sponsor in Alabama had not only a similar idea, but a more elaborate one. He wanted to hold a reunion of all his former Mu Alpha Theta members. The big difference between this teacher and me is that my Alabama colleague has turned his idea into reality.

Edwin Guthrie, sponsor at West Point High School in Cullman, AL, had a dream of bringing together all former Mu Alpha Theta members in one big reunion. Having served as the chapter's advisor since its inception in 1972, Guthrie had an enormous undertaking ahead of him. Toying with the reunion notion for some time, he finally determined to try it. With the help of current members, he selected December 23 for the reunion, hoping to find as many graduates as possible at home for the holidays. In retrospect, Guthrie's choice was a good one.

Assisting in the formidable task were present Mu Alpha Theta officers Andrea Morton, Christy Quattlebaum, Darlene Kent, Tina Coots, and Joey Skinner, Mu Alpha Theta members, other students, and parents. Invitations and information forms were mailed to about 150 students. From this number, 65 returned their forms and 67 Mu Alpha Theta people attended the reunion.

The present Mu Alpha Theta membership went all out to welcome back their Mu Alpha Theta alumni. They provided a guest register, so that the alumni could "sign in," providing a lasting record. Decorations were everywhere, including poinsettias supplied by a local florist. The members served soft drinks, finger foods, and cakes emblazoned with the Mu Alpha Theta emblem. They also provided everyone with blue napkins with blue lettering, imprinted for the occasion.

The program was carefully planned to work in both current and graduated members. Also in attendance were the school principal and the superintendent. They briefly addressed the gathering, and (we're told) "praised those present for their accomplishments in math competitions tions and for the subsequent successes they have enjoyed." Later, each Mu Alpha Theta introduced himself and made a few brief remarks, if he chose to do so. Ample time was provided for meeting classmates and socializing.

According to Edwin Guthrie, all seemed to enjoy the time spent together. Many suggested that such a reunion be an activity on a regular basis. A considerable number of former students expressed appreciation for the opportunities offered by Mu Alpha Theta affiliation, and indicated that such experiences had been invaluable. Several made reference, we're told, to problem solving skill acquired in competitive events having proven beneficial in college and later as they assumed various roles in society.

This success, to which many alluded, can be linked to their high school days. Many had been active in interscholastic math competitions during their years at West Point High School. In fact, in the fifteen years of Mu Alpha Theta at West Point, students have won approximately

PLAN NOW ... 1990 CONVENTION IN ILLINOIS

300 team trophies and awards as well as a greater number of individual awards. Constructing oak trophy cases in numbers suffient to display the trophies, the School proudly featured an exhibit of awards on reunion night.

On sidelight on Mu Alpha Theta graduates at this institution: of 65 information forms returned, 25% of those who responded had earned or were working on degrees in engineering, and another 35% had earned or were working on degrees in other math-related areas. The vast majority of those who replied had continued their formal education beyond high school level.

This reunion of mathematical minds ... a fascinating insight as to how the power of mathematics can charge our lives with energy ... to extraordinary results.

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MU ALPHA THETA

The Mathematical Log is the official publication of Mu Alpha Theta, national high school and junior college mathematics honor society and mathematics club federation. Mu Alpha Theta, founded in 1957 by Richard and Josephine Andree, is co-sponsored by the Mathematical Association of America (MAA) and the National Council of Teachers of Mathematics (NCTM). The Mathematical Log is published quarterly, in February, April, October, and December. Correspondence may be directed to specific editors or to Mu Alpha Theta National Office, 601 Elm Ave., Rm. 423, Norman, OK 73019.

Math of Investment Yields Social Security Insights

by
Ali R. Amir-Moéz
Mathematics Editor

Everyone may enjoy computing the amount paid for social security and may also find out how much it amounts to with a modest interest. We shall study the simple mathematics involved which amounts to the study of geometric progression and a few other ideas.

This note is a review of compound interest which may interest a few young people to study and play with it. When something of interest is in the background, one would put more effort in it.

1. The Compound Interest: Let the Social Security contribution for each year be a. Let the rate of interest be r and compounded k times per year. Then we observe that for each period the rate will be $\frac{r}{k}$. Now the end of the first period we shall have

$$a + a\frac{r}{k} = a\left(1 + \frac{r}{k}\right) = a_1. \tag{1}$$

At the end of the second period we shall have

$$a_2 = a_1\left(1 + \frac{r}{k}\right) = a\left(1 + \frac{r}{k}\right)^2. \tag{2}$$

So after n years we will have kn periods and the amount for the n years will be

$$A_n = a\left(1 + \frac{r}{k}\right)^{kn}. \tag{3}$$

So after n years we have the sum

$$S = A_1 + A_2 + \ldots + A_n. \tag{4}$$

One observes that S is the sum of n terms of the geometric progression

$$\begin{aligned} S &= a\left(1 + \frac{r}{k}\right)^k + a\left(1 + \frac{r}{k}\right)^{2k} + \ldots + a\left(1 + \frac{r}{k}\right)^{nk} \\ &= a\left[\left(1 + \frac{r}{k}\right) + \ldots + \left(1 + \frac{r}{k}\right)^{nk}\right]. \end{aligned} \tag{5}$$

For simplicity let $\left(1 + \frac{r}{k}\right)^k = p$. Then (5) will be

$$S = a[p + \ldots + p^n], \tag{6}$$

and multiplying both sides by p we get

$$pS = a[p^2 + \ldots + p^{n+1}]. \tag{7}$$

Subtracting (6) from (7), we obtain

$$PS - S = (p-1)S = a(p^{n+1} - p) = ap(p^n - 1). \tag{8}$$

Consequently, we get

$$S = a\frac{p(p^n - 1)}{p - 1}. \tag{9}$$

So for $p = \left(1 + \frac{r}{k}\right)^k$ we obtain

$$S = a\frac{\left(1 + \frac{r}{k}\right)^k\left[\left(1 + \frac{r}{k}\right)^{nk} - 1\right]}{\frac{r}{k}} \tag{10}$$

or

$$S = \frac{ak}{r}\left(1 + \frac{r}{k}\right)^k\left[\left(1 + \frac{r}{k}\right)^{nk} - 1\right]. \tag{11}$$

Now let us give a modest example. Suppose a person has an income of $10,000 per year. Usually up to $37,800 one pays 6.7% of his or her income for social security tax. (Note that this is somewhat out of date.)

So the person of our example pays $670 per year, that is, if he or she remains with the same income. One is aware that a high capital gets large interest, but we suppose that the person in our example receives the interest of 10% once a year. If he or she starts working at age of sixteen and pays 35 years of social security tax and it is only compounded once a year, that person by (11) will have

$$\begin{aligned} S &= \frac{670}{.10}\left\{1.1[1.1]^{35} - 1\right\} \\ &= (67)(1.1)^{36} - 1.1 = 199,744.96. \end{aligned}$$

Imagine what a person with average income will have to his or her credit.

2. The Continuous Compounding: Let us consider (3), that is

$$A_n = a\left(1 + \frac{r}{k}\right)^{kn}. \tag{12}$$

This can be written as

$$A_n = a\left[\left(1 + \frac{r}{k}\right)^{\frac{k}{r}}\right]^{nr}. \tag{13}$$

When the continuous compounding is considered, k becomes very large. In this case we say k approaches infinity. In symbol we write $k \to \infty$. (Even though this may sound very impressive, it does not add much to the interest.) It is well known that

$$\lim_{k\to\infty}\left(1 + \frac{r}{k}\right)^{\frac{k}{r}} = e. \tag{14}$$

Usually e is approximated by 2.7, but one may consider more decimals such as

$$e \cong 2.718281828459045.$$

From (14) we get

$$A_n = ae^{nr} \tag{15}$$

Then for (5) we get

$$\begin{aligned} S &= A_1 + \ldots + A_n = a(e^r + e^{2r} + \ldots + e^{nr}) \\ &= a\frac{e^{(n+1)^r - e^r}}{e^r - 1}. \end{aligned} \tag{16}$$

Let us now compute the value of our example using (16). We get

$$S = 670\frac{e^{36(.1)} - e^{.1}}{e^{.1} - 1} \cong 226,111.49.$$

Indeed, it is quite easy to call a bank and ask the question about how much would be the amount when so much a year is put in a savings account with a certain rate. But it is more interesting to compute one's own amount. Imagine if one puts the 7% of an income of $37,800 per year in the social security and consider a 12% interest compounded continuously, how much does one have after thirty years of work. Why don't you try (16) and use a good hand calculator?

Now let us ask, *"Where does the money go?"* The Social Security payment is quite poor including other benefits such as medicare, etc. Enrolling in medicare costs also another $14.60 a month which is $175.2 a year. (This amount is also somewhat out of date.)

3. Questions: Indians received *twenty four* dollars for Manhattan Island in the year 1576. If they had invested that money, by the year 1920, it would have become 8,271,157,608. What rate would bring that much money if it were compounded *continuously*?

What would be the rate if it were compounded once a year?

What would be the rate if it were compounded four times a year?

Aim for Higher Goals Andree Winner Urges

"I want to tear down the mental wall that students may have built against mathematics and show them how useful a sound knowledge of mathematics can be."

Tammy Kirkland, Mu Alpha Theta president at East Central Community College, Decatur, MS, penned those words as a candidate for the Richard V. Andree "future teacher" award.

As Andree Award winner getting ready to travel to Mu Alpha Theta national convention in Tampa, Tammy shared these thoughts which she asked be shared with Mathematical Log readers:

"I would like to challenge all Mu Alpha Theta members to continue your interest in mathematics and to continue to excel in your coursework.

"Never be satisfied, always aim for higher goals--but all the while, remain proud of yourself and your accomplishments."

Tammy graduated from Neshoba Central High School with an outstanding student record and special awards in mathematics, science, and band, but with no experience with Mu Alpha Theta, the high school having had no chapter. She was welcomed into the community college chapter on the basis of her first-term grades, and was elected chapter president in her second year. Tammy also was Student Education Association president, Phi Theta Kappa vice-president, scholar's bowling team captain, and President's Council member. She won the Freshman Mathematics Award, 1989 ECCC Outstanding Student Award, and was named to the National Dean's List and All-American Scholar and Presidential Scholar. She plans to continue her studies at Mississippi State University, majoring in Mathematics and minoring in Education, in preparation for her career as mathematics teacher.

In her Andree Award essay presenting her rationale for choosing mathematics teaching as a future career, Tammy identified it as a lifetime ambition--to teach--coupled with a desire to share her favorite subject.

Tammy wrote:

"As a young child, like most little girls, I loved to pretend that I was a teacher. I would seat my dolls all around me and teach them all that I had learned, meticulously imitating my own teachers. I often dreamed of the day when I would have my own classroom with real students. However, unlike most young girls, my dream has not changed.

"I have always wanted to become a teacher. I have often been told that I should become a doctor, a lawyer, or even a scientist. Some have remarked that it seems a waste of my potential to become a teacher; yet, I can think of no better way to use my abilities than by helping others expand their minds.

"Throughout school, mathematics has always been my favorite subject. In fact, I recall a time when I was only in grammar school that after our mathematics teacher explained a new topic and assigned work for us to do, several of my friends would call me over to re-explain the topic. They always told me that I had a special ability to explain topics in a way that they could better understand. This became a trend which continued throughout high school. Mathematics comes so naturally to me that I feel my capacity for it must have been inborn.

"I feel that mathematics is the most important subject in the school curriculum. A person needs a strong mathematics background no matter what he chooses to do in life. As our society becomes more and more technological, that need for mathematical knowledge will become more imperative.

"Yet so many people perform poorly in mathematics. Some mistakenly think that mathematics is boring. On the contrary, mathematics can be very exciting. One can find so many ways to be creative in teaching math. I do not want my students to become bored with mathematics. I want to tear down the mental wall that students may have built against mathematics and show them how useful a sound knowledge of mathematics can be. Once students can see a purpose for learning mathematics, I feel that they will develop a great enjoyment of it.

"In summary, I want to become a mathematics teacher because I feel the nation needs more dedicated, innovative teachers for its young people. I want to instill in my students a love for mathematics as well as a love for learning."

Tammy, who tells The Log how much she has enjoyed being a part of Mu Alpha Theta during her community college years, encourages students who are Considering a career in mathematics teaching to submit an Andree Award application. Any who wish to contact Tammy may do so at Route 2, Box 218, Philadelphia, MS 39350. Tammy's Mu Alpha Theta sponsor at East Central Community College has been Mrs. Lois McMullan.

diaLogue

with Log Editor Don Allen

A pleasant if improbable evening with hyenas, eagles, and snakes--mutually devouring critters--has left us with a feeling of some satisfaction, but with more good questions than we started with, as open-ended math problems frequently will.

The hyenas, eagles, and snakes derive, as you might suspect, from the Swaziland math competition, featured in part in February's Log (page 8). Our evening with the African fauna we owe to Sandra Hightower of Skyline High School, Dallas, TX--who wrote requesting--not unreasonaby--the answers to the trio of Swaziland questions.

The first two questions were distinctive but routine, and we'll treat them accordingly. The third, involving animal populations, "opens up" intriguingly, so we'll both provide answers and suggest directions for further investigation.

Here are the three Swaziland problems, with answers and rationales.

1. If it is raining in Mbabane at midnight, will Manzini have sunny weather 72 hours later?

A classic "trick question" in African dress. The sun may never have set on the British Empire (Swaziland, a constitutional monarchy, has long British connections), but any given part of it sure knew night and day! Midnight plus 72 hours equals midnight, so no sun in Manzini!

2. Seven good friends dine in the same restaurant. All are eating there today. However, they do not all eat there every day. The first eats there every day, the second every second day, the third every third day, ..., and the seventh every seventh day. How Many days from today will they all meet in the restaurant?

Let's talk about it.

The seven certainly will be together again in 1 x 2 x 3 x 4 x 5 x 6 x 7, or 5040 days (something over 13 years), or any multiple of 5040 days.

But, in fact, they will be together much sooner. We need not the product but the least common multiple of 1 through 7, since the LCM will be less than the product due to 2, 4, and 6 and 3 and 6 having common factors.

(See "diaLogue," page four)

Group 'Power Question'

FROM PAGE ONE

P4. Find the sides of this P4.

B. Find a P4 with two sides equal and with d = 25 that is different from the answer to part IIA. [Note: Two Pn's are not considered different if their sides are equal, but in a different order.]

C. Show that a Pn must exist for all integers $n \geq 3$. [This may be done by describing how to create such a Pn.]

III.

A. For the P3:(a,b,d), $d^2 = a^2 + b^2$. Prove that for the P4:(a,b,c,d), $d^2 > a^2 + b^2 + c^2$.

B. Given the P4:(a,b,c,d). Prove that if d > 2, then d must be composite.

C. If all the diagonals of a Pn are integers, we will call it "Super Pythagorean" and denote it by $\overline{Pn}$.

1. Show that the area of any $\overline{P4}$ must be an integer. [Hint: One approach might be to first show that the area of any $\overline{P4}$ must be rational, and its perimeter must be even.]

2. Assuming that the area of every $\overline{P3}$ and every $\overline{P4}$ is an integer, show that (for all n > 4) the area of every $\overline{Pn}$ must be an integer. [You may do this part even if part IIIC1 has not been completed.]

diaLogue

FROM PAGE THREE

The required LCM is 3 x 4 x 5 x 7, or 420. The friends will reassemble in 420 days and in every multiple of 420 days.

Now the hyenas, eagles, and snakes:

3. In a field are some hyenas, eagles, and snakes. Every morning every hyena eats one eagle. At mid-day every snake eats one hyena. In the evening every eagle eats one snake. At the end of the third day there was one eagle left and no others. How many of each were there at the beginning of the first day?

Read carefully, draw up a table, watch for number errors, and you'll not go wrong: 13 hyenas, 19 eagles, and 9 snakes. Work forward as a check.

* * *

To see how the Swaziland problem has potential for "opening up," extend your tabulation backward in time for, say, seven days instead of three days, and give thought to the sequences involved. (The "field" gets a bit crowded, but as with counting ancestors back to the nth generation, that doesn't spoil the math of the theoretical problem!) Here's a tabulation through 21 meals, to an initial population of 872 snakes, 1278 hyenas, and 1873 eagles--4023 animals in all. Note how the entries in columns for specific species, for example the "9, 13, 19" of the original problem, turn up [underscored] in the totals column as well. That "1, 2, 3, 4, 6, 9, ..." "total" sequence seems to say it all, but exactly how may be less than clear.

The sequence, though with three "components," serves to recall the classic Fibonacci sequence (1, 1, 2, 3, 5, ...) and its "rabbits" problem, from which so much good mathematics has arisen.

DAY OF 7	DAY OF 3	FEEDING	NUMBER OF ANIMALS SURVIVING FEEDING snakes (s)	hyenas (h)	eagles (e)	total
(0)			872	1278	1873	4023
1		i	872	1278	595	2745
		ii	872	406	595	1873
		iii	277	406	595	1278
2		i	277	406	189	872
		ii	277	129	189	595
		iii	88	129	189	406
3		i	88	129	60	277
		ii	88	41	60	189
		iii	28	41	60	129
4		i	28	41	19	88
		ii	28	13	19	60
	(0)	iii	9	13	19	41
5	1	i	9	13	6	28
		ii	9	4	6	19
		iii	3	4	6	13
6	2	i	3	4	2	9
		ii	3	1	2	6
		iii	1	1	2	4
7	3	i	1	1	1	3
		ii	1	0	1	2
		iii	0	0	1	1

* * *

A different route to exploring the potential of the Swaziland problem would be to retain the "three days, nine meals" feature while allowing the numbers of surviving creatures to vary. The "general case" might involve s surviving snakes, h hyenas, and e eagles.

Following through with this idea, we "open up" as follows:

Let the numbers of snakes, hyenas, and eagles at the end of a given day be, respectively, s, h, and e.

Let the numbers of snakes, hyenas, and eagles at the start of that day, or at the end of the preceding day, be, respectively, S, H, and E.

Then:

--prior to the evening feeding, there were s + e snakes, h hyenas, and e eagles;

--prior to the mid-day feeding, there were s + e snakes, h + s + e hyenas, and e eagles;

--prior to the morning feeding, there were s + e snakes, h + s + e hyenas, and e + (h + s + e), or 2e + h + s eagles.

So $S = s + e$, $H = h + s + e$, and $E = 2e + h + s$.

Let the number of snakes, hyenas, and eagles at the start of the preceding day be, respectively, S_2, H_2, and E_2.

Then, extending the above reasoning,

--$S_2 = S + E = (s + e) + (2e + h + s) = 2s + h + 3e$;

--$H_2 = H + S + E = (h + s + e) + (s + e) + (2e + h + s) = 2h + 3s + 4e$;

--$E_2 = 2E + H + S = 2(2e + h + s) + (h + s + e) + (s + e) = 6e + 3h + 4s$.

Finally, let the number of snakes, hyenas, and eagles at the start of the day before that be, respectively, S_3, H_3, and E_3.

Further extending the above reasoning,

--$S_3 = S_2 + E_2 = (2s + h + 3e) + (6e + 3h + 4s) = 6s + 4h + 9e$;

--$H_3 = H_2 + S_2 + E_2 = (2h + 3s + 4e) + (2s + h + 3e) + (6e + 3h + 4s) = 6h + 9s + 13e$;

--$E_3 = 2E_2 + H_2 + S_2 = 2(6e + 3h + 4s) + (2h + 3s + 4e) + (2s + h + 3e) = 19e + 9h + 13s$.

That is, in effect: if at the end of the third day (in the Swaziland problem) there had been s snakes, h hyenas, and e eagles, then at the beginning of the first day there would have been 6s + 4h + 9e snakes, 6h + 9s + 13e hyenas, and 19e + 9h + 13s eagles.

(continued, page six)

LET'S HEAR FROM YOU!

Photos, cartoons, news items, personal notes, problems you pose, ideas you "write up," all make it distinctively your Mathematical Log. Resolve to keep your interests--and your chapter--in the news.

Math of Investment Yields Social Security Insights

by
Ali R. Amir-Moéz
Mathematics Editor

Everyone may enjoy computing the amount paid for social security and may also find out how much it amounts to with a modest interest. We shall study the simple mathematics involved which amounts to the study of geometric progression and a few other ideas.

This note is a review of compound interest which may interest a few young people to study and play with it. When something of interest is in the background, one would put more effort in it.

1. The Compound Interest: Let the Social Security contribution for each year be a. Let the rate of interest be r and compounded k times per year. Then we observe that for each period the rate will be $\frac{r}{k}$. Now the end of the first period we shall have

$$a + a\frac{r}{k} = a\left(1+\frac{r}{k}\right) = a_1. \qquad (1)$$

At the end of the second period we shall have

$$a_2 = a_1\left(1+\frac{r}{k}\right) = a\left(1+\frac{r}{k}\right)^2. \qquad (2)$$

So after n years we will have $\frac{kn}{n}$ periods and the amount for the n years will be

$$A_n = a\left(1+\frac{r}{k}\right)^{kn}. \qquad (3)$$

So after n years we have the sum

$$S = A_1 + A_2 + \ldots + A_n. \qquad (4)$$

One observes that S is the sum of n terms of the geometric progression

$$\begin{aligned} S &= a\left(1+\frac{r}{k}\right)^k + a\left(1+\frac{r}{k}\right)^{2k} + \ldots + a\left(1+\frac{r}{k}\right)^{nk} \qquad (5)\\ &= a\left[\left(1+\frac{r}{k}\right) + \ldots + \left(1+\frac{r}{k}\right)^{nk}\right]. \end{aligned}$$

For simplicity let $\left(1+\frac{r}{k}\right)^k = p$. Then (5) will be

$$S = a[p + \ldots + p^n], \qquad (6)$$

and multiplying both sides by p we get

$$pS = a[p^2 + \ldots + p^{n+1}]. \qquad (7)$$

Subtracting (6) from (7), we obtain

$$PS - S = (p-1)S = a(p^{n+1} - p) = ap(p^n - 1). \qquad (8)$$

Consequently, we get

$$S = a\frac{p(p^n-1)}{p-1}. \qquad (9)$$

So for $p = \left(1+\frac{r}{k}\right)^k$ we obtain

$$S = a\frac{\left(1+\frac{r}{k}\right)^k\left[\left(1+\frac{r}{k}\right)^{nk} - 1\right]}{\frac{r}{k}} \qquad (10)$$

or

$$S = \frac{ak}{r}\left(1+\frac{r}{k}\right)^k\left[\left(1+\frac{r}{k}\right)^{nk} - 1\right]. \qquad (11)$$

Now let us give a modest example. Suppose a person has an income of \$10,000 per year. Usually up to \$37,800 one pays 6.7% of his or her income for social security tax. (Note that this is somewhat out of date.)

So the person of our example pays \$670 per year, that is, if he or she remains with the same income. One is aware that a high capital gets large interest, but we suppose that the person in our example receives the interest of 10% once a year. If he or she starts working at age of sixteen and pays 35 years of social security tax and it is only compounded once a year, that person by (11) will have

$$\begin{aligned} S &= \frac{670}{.10}\{1.1[1.1]^{35} - 1\}\\ &= (67)(1.1)^{36} - 1.1 = 199{,}744.96. \end{aligned}$$

Imagine what a person with average income will have to his or her credit.

2. The Continuous Compounding: Let us consider (3), that is

$$A_n = a\left(1+\frac{r}{k}\right)^{kn}. \qquad (12)$$

This can be written as

$$A_n = a\left[\left(1+\frac{r}{k}\right)^{\frac{k}{r}}\right]^{nr}. \qquad (13)$$

When the continuous compounding is considered, k becomes very large. In this case we say k approaches infinity. In symbol we write $k \to \infty$. (Even though this may sound very impressive, it does not add much to the interest.) It is well known that

$$\lim_{k\to\infty}\left(1+\frac{r}{k}\right)^{\frac{k}{r}} = e. \qquad (14)$$

Usually e is approximated by 2.7, but one may consider more decimals such as

$$e \cong 2.718281828459045.$$

From (14) we get

$$A_n = ae^{nr} \qquad (15)$$

Then for (5) we get

$$\begin{aligned} S &= A_1 + \ldots + A_n = a(e^r + e^{2r} + \ldots + e^{nr}) \qquad (16)\\ &= a\frac{e^{(n+1)r} - e^r}{e^r - 1}. \end{aligned}$$

Let us now compute the value of our example using (16). We get

$$S = 670\frac{e^{36(.1)} - e^{.1}}{e^{.1} - 1} \cong 226{,}111.49.$$

Indeed, it is quite easy to call a bank and ask the question about how much would be the amount when so much a year is put in a savings account with a certain rate. But it is more interesting to compute one's own amount. Imagine if one puts the 7% of an income of \$37,800 per year in the social security and consider a 12% interest compounded continuously, how much does one have after thirty years of work. Why don't you try (16) and use a good hand calculator?

Now let us ask, *"Where does the money go?"* The Social Security payment is quite poor including other benefits such as medicare, etc. Enrolling in medicare costs also another \$14.60 a month which is \$175.2 a year. (This amount is also somewhat out of date.)

3. Questions: Indians received *twenty four* dollars for Manhattan Island in the year 1576. If they had invested that money, by the year 1920, it would have become 8,271,157,608. What rate would bring that much money if it were compounded *continuously*?

What would be the rate if it were compounded once a year?

What would be the rate if it were compounded four times a year?

CONVENTIONS DO DIFFER

Mu Alpha Theta, as we all know, has many facets. The local chapter meeting, its program informal, its welcome genuine, its enthusiasm infectious, is where much of the learning and growing takes place, where confidence is built, horizons are broadened, friendships are made. The contests, the math fairs, the regional meetings, are a logical, and most useful, extension. But there's nothing in Mu Alpha Theta quite like a national convention--where everything "comes together"--and a national convention is something that we think every member should experience at least once.

National conventions of Mu Alpha Theta are hosted by local chapters, and each one is distinctly different, reflecting the people, the interests, and the ways of doing things of the local group or groups that are welcoming up to 800 or more young people and their teacher-sponsors from across the nation and beyond.

Most Mu Alpha Theta members who are reading these words either will be at convention--our 19th in Tampa where the Back-to-School Log and Timbers are predistributed--or at school in their chapters, where thought might be given to attendance and full participation in our 20th national convention in Illinois.

Two things that all agree upon: That convention attendance calls for advanced planning and hard work, and that, in the end, it's all more than worthwhile.

We stress this, because, figures show, more than 95% of Mu Alpha Theta members and their sponsors never make it to a national convention.

Those of us who have been to most of the national conventions to date--no one has been to all!--are struck by both the diversity, reflecting local attractions, interests, and ways of doing things, as well as by the predictability--that groups will get to know one another, lasting friendships have their beginning, and that many will gain important insights into the subject and its possibilities for a career and a way of life.

It's good to spend days With hundreds of active, well-behaved, committed young people ... none of whom feels moved to say, "I never could do mathematics, and I can't see what use it will be." A positive attitude, like a negative one, can be highly infectious, and the aftermath of a top convention can be a time for clarifying thoughts.

Our cherished Mu Alpha Theta convention memories include rich dialogue with sponsors, parents, guests, and young people, especially young people, in diverse, on occasion improbable, locations: Georgia Tech, Tulane, and Wisconsin dorms, Oklahoma City and Seattle science centers, Seattle Aquarium, Disneyland, St. Louis' Six Flags and Gateway Arch, Dollywood, and aboard Kamehameha's yellow school buses, savoring the wraparound views from Diamond Head and Waikiki to Pearl Harbor. Each convention has had aspects that were highly distinctive. We recall with affection, our first convention, at Stevens Point, with the accent on home-grown hospitality, and a picnic by the river in a town park. Groups mingled, friendships were made, perhaps to a greater extent than would have been possible had Mu Alpha Theta been but part of a larger crowd.

Top speakers (including students!) have become a convention tradition. That leading figures from the mathematics community and related fields, take time for Mu Alpha Theta, says much. We particulary recall TV weatherman George Fischbeck ("you have to fail to succeed") at UCLA, aerospace consultant James Nikkel at Honolulu, and National Hurricane Center's Robert Sheets (with incredible slides) at University of Miami. Further, Harold Jacobs at Georgia Tech and Zal Usiskin at Oklahoma "talked mathematics" as few have heard it spoken, as indeed did Mu Alpha Theta's own Harold Huneke, Eugene Smith, Richard Rhodes, Harry Ruderman, Paul Foerster, and the late Richard Andree.

Plan now, we urge, for Illinois Convention 1990! H.D.A.

diaLogue

... FROM PAGE FOUR

have been 6s + 4h + 9e snakes, 6h + 9s + 13e hyenas, and 19e + 9h + 13s eagles.

In the specific instance of the problem, $s = 0$, $h = 0$, and $e = 1$, giving initial values of 9 snakes, 13 hyenas, and 19 eagles.

* * *

Patterns, finite differences, recursion relations--if not wildlife management and counselling of self-endangered species!--interrelate in that modest but uncommonly instructive problem. If you've looked at all into recursion relations, or are quick in spotting a pattern, the 1, 2, 3, 4, 6, 9, 13, ..., sequence of animal totals has initial terms 1, 2, 3, then subsequent terms formed by increasing the last term by the one two terms before it: in the notation of recursion relations, if the k^{th} term (denoting the number surviving after the k^{th} feeding ago) is u_k, then:

$u_1 = 1$, $u_2 = 2$, $u_3 = 3$; and $u_k = u_{k-1} + u_{k-3}$, for $k \geq 4$.

Numbers of each species can be derived from total numbers surviving in straightforward ways.

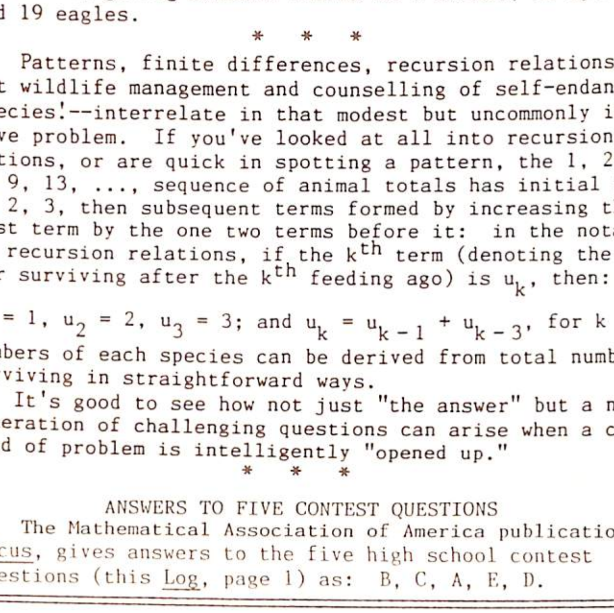
It's good to see how not just "the answer" but a new generation of challenging questions can arise when a certain kind of problem is intelligently "opened up."

* * *

ANSWERS TO FIVE CONTEST QUESTIONS

The Mathematical Association of America publication, Focus, gives answers to the five high school contest questions (this Log, page 1) as: B, C, A, E, D.

CROSS-MATH...



A CHALLENGE ESCALATES! Here, for those who like their challenges numerical, is the completed grid for the Cross-Math Challenge in April's Tall Timbers supplement, #22. The key to success was care with "priority of operations," multiplying before adding in, say, 2 + 3 x 4; taking the power before the product in, say, 3 x 5². Now for the next phase of the challenge--"cracking" the 115-letter cipher message concealed by the digits reproduced above!

Mu Alpha Theta
Officers

1981	Pres.	Roger Gunn
	v.p.	Randy Russell
	sec.	Vikki Jenkins
	Tr.	Terri Russell
	SocC	Stella Posey
1982	Pres.	Greg Henley
	v.p.	Ken Sims
	sec.	Ken Wallace
	Tr.	Larry Germany
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	Rep	Ramona Pullin
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	Sec	Connie Myers
	Treas	Connie Myers
	Social	Kerry Wilcher, Stephanie Gainey
	Report	Lisa Risher
1990	Pres.	Kevin Winstead
	V. P.	Jim Holder
	Sec/Tr.	Robin Cole
	Report.	Dwayne McNair
	Soc. C.	Kay Vaughn
		Kim Griffin

MU ALPHA THETA
1991-1992 Officers

President:	Tam Cooper
Vice-President:	Richie Rivers
Secretary:	Jason Smith
Reporter:	Tracy Blackburn
Social Chairpersons:	Patsy Harris and Cathie Jones

MU ALPHA THETA
1992-1993 Officers

President: Kim Thornton

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Secretary: Amy Shows

Reporter: Sonja Quinn

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MU ALPHA THETA
1993-1994 Officers

President:	James Crapse
Vice-President:	
Secretary:	Kim Reel
Reporter:	
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MU ALPHA THETA
1994-1995 Officers

President:	Kris Rigdon
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Secretary:	Mike Hurst
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Program Committee:	John Ferguson Hartman Holliman Joey Thrash
Project Committee:	Paul Killen Misty Savell Timothy Thompson
Social Committee:	Jacqueline Cain Carla Glenn Susan George

MU ALPHA THETA
1995-1996 Officers

President:	Chris Gomillion
Vice-President:	Brad Gressett
Secretary:	Valerie Buggs
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Program Committee:	Shannon Goss Ty Nowell Matt Skinner Ryan Smith
Project Committee:	Jill Kirkland Amy Phillips Staci Strebeck
Social Committee:	Mandy Goldman Kristi Gordon Elaine Blount

MU ALPHA THETA
1996-1997 Officers

President:	Olivia Hamil
Vice-President:	Bart Gregory
Secretary:	Andy Sistrunk
Reporter:	Amy Boatner
Program Committee:	Jonathan Sharp, Chair Amy Moore Melanie Webb
Project Committee:	Karen McRae, Chair Justin Griffin Lee Rigdon
Social Committee:	Stephanie Taylor, Chair Karen Persons Kim Tew

MU ALPHA THETA
1997-1998 Officers

President:	Krisan L. Risher
Vice-President:	
Secretary:	Ginger Greer
Reporter:	
Social Chairpersons:	

MU ALPHA THETA
1998-1999 Officers

President: Laura Hope Faulk

Vice-President: Scicile Wilkerson

Secretary: Dana Dye

Reporter: Ragan Mitchell

Program Committee: Cody Fisher, B. J. Morrow, Jodi Stribling

Project Committee: Vanessa Sanders, Phillip Smith, Brandi Duncan, Candice Butler

Social Committee: Anita Moss, Betina Latiker, Angela Hays

MU ALPHA THETA
1999-2000 Officers

President:	Kristi French
Vice-President:	Kelly Tucker
Secretary:	Craig McRae
Reporter:	Connie Tew

Project Committee Chairman: Ginny Rigby

Social Committee Chairman: Jennifer Bounds

MU ALPHA THETA
2000-2001 Officers

President:	Ashley Jones
Vice-President:	Alicia Purvis
Secretary:	Michael Thompson
Reporter:	Matt Baldridge

Project Committee Chairman: Lisa Adcock

Social Committee Chairman: Emily Boggan

MU ALPHA THETA
2001-2002 Officers

President:	Allison Latham
Vice-President:	Samantha Styburski
Secretary:	Jason Guraedy
Reporter:	Matt Davis

Mu Alpha Theta
601 Elm Avenue, Room 423
Norman, Oklahoma 73019

Special To

Norman, OK, February 16, 1984---Fourteen students at East Central Junior College were honored recently by election to Mu Alpha Theta, international high school and junior college mathematics club.

The announcement was made by Dr. Harold V. Huneke, national secretary-treasurer, who is a professor of mathematics at the University of Oklahoma where the national office is located.

Only those schools with excellent mathematics programs can earn membership in the club since all courses in mathematics and the qualifications of the mathematics faculty and students are examined in detail by the club's Governors and National Officers.

To be eligible for membership, minimum requirements are that a student must have completed with distinction (at least a B average) at least four semesters of college preparatory mathematics and be enrolled in the fifth semester.

"Membership in Mu Alpha Theta is the highest honor possible for a high school or junior college student of mathematics," Dr. Huneke said. "Club activities consist of work in areas of mathematics not usually covered in the classroom."

Mu Alpha Theta was founded in 1957 at the University of Oklahoma and has grown to more than 1,200 active chapters in most of the states and seven foreign countries.

The club is co-sponsored by the Mathematical Association of America and the National Council of Teachers of Mathematics and has attracted the attention of top mathematics scholars in this country and abroad.

REQUIREMENTS FOR ADMISSION

ALGEBRA A

TRIG A,B

CAL I A,B

CAL IIA,B,C

CALIII A,B,C

CAL IV A,B,C

East Central Junior College
Decatur, Mississippi
~~October 26, 1981~~

March 11, '82

This is your invitation to join the East Central Junior College chapter of Mu Alpha Theta, a junior college mathematics club. The Club, which is co-sponsored by the Mathematical Association of America, and the National Council of Teachers of Mathematics, is a non secret organization whose purpose is to stimulate interest in mathematics by providing public recognition of superior mathematical scholarship.

Your ~~invitation depends on the petition to have~~ a ch~~arter at East Central Junior~~ College. Mu Alpha ~~Theta, the Mathematics Club will be the~~ organiza~~tional na~~me. An initdation fee of $3.00 per full member shall be paid to the secretary-treasurer, $2.00 of this fee will be sent to the national secretary-treasurer, where-upon you shall be issued a membership certificate andyour name will be placed on the official roll.

Congratulations for having qualified for such an honor! Please see me and let me know your wishes as to membership in Mu Alpha Theta.

Sincerly,

Shelby L. Harris

Dr. Shelby L. Harris,
Sponsor, Mu Alpha Theta

SLH;dh

MU ALPHA THETA
East Central Junior College
Decatur, Mississippi

This is your invitation to join the East Central Junior College Chapter of Mu Alpha Theta, a mathematics club. The club, which is cosponsored by the Mathematical Association of America and the National Council of Teachers of Mathematics, is a non-secret organization whose purpose is to increase interest in mathematics by providing public recognition of superior mathematical scholarship.

An initiation fee of $4.00 per full member shall be paid, $2.00 of which will be sent to the national office, where-upon you shall be issued a membership certificate and your bame will be placed on the nation roll.

Congratulations for having qualified for such an honor! Please see me and let me know your wishes as to membership in Mu Alpha Theta.

Sincerely

Dr. Shelby L. Harris, Sponsor
Mu Alpha Theta

SLH;dh

NATIONAL HIGH SCHOOL AND JUNIOR COLLEGE

Mathematics Club

Mu Alpha Theta (M A Th)

Petition for Charter

Mu Alpha Theta
601 Elm Avenue, Room 423
Norman, Oklahoma 73019

Submit Original Plus (10) Copies on Plain Paper

The undersigned hereby petitions that EAST CENTRAL JUNIOR COLLEGE
(Name of School)

DECATUR, MISSISSIPPI 39327
(Address of School) (City) (State) (Zip Code)

be granted a charter. The following information is submitted to guide the Governing Council in determining the eligibility of the school.

1. On separate sheets, list a syllabi of mathematics courses and text books used for the sophomore, junior and senior years of the college preparatory program in your school. The syllabi should contain a brief outline of the topics covered.

2. Approximate enrollment in sophomore, junior and senior levels of the college preparatory program in mathematics for the past three years, and the total number of students graduated in all areas in each of these years.

Year	Sophomore Math	Junior Math	Senior Math	Total all students graduated

3. Name of teacher (not necessarily the same as the sponsor) employed for at least two semesters whose primary teaching field is mathematics and who has completed an undergraduate mathematics major or its equivalent.

Shelby L. Harris

Major field of the teacher Mathematics Highest degree earned Doctor of Educatio

in 1974 (Year) at University of Southern Mississippi (Name of College or University)

List courses by title this teacher has taken above calculus: (If more than six courses are involved list only six)

Adv Calculus I, II, Matrix Algebra, Modern Alg I, II, Adv. Algebra,

4. Name of faculty sponsor Dr. Shelby L. Harris

5. Attached is a certificate order blank giving names of all full-member initiates. The Charter fee of $2.00, plus ~~$1.00~~ 2.00 initiation fee for each full member, totaling $ 52.00, is enclosed.

Shelby L. Harris
(Signature of Faculty Sponsor)

This petition is approved by the principal of the school.

Charles V. Wright, President
(Signature of Principal or Chief Administrative Officer)

10-26-81

NATIONAL HIGH SCHOOL AND JUNIOR COLLEGE

Mathematics Club

Mu Alpha Theta (M A Th)

Petition for Charter

Mu Alpha Theta
601 Elm Avenue, Room 423
Norman, Oklahoma 73019

Submit Original Plus (10) Copies on Plain Paper

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(Name of School)

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2. Approximate enrollment in sophomore, junior and senior levels of the college preparatory program in mathematics for the past three years, and the total number of students graduated in all areas in each of these years.

Year	Sophomore Math	Junior Math	Senior Math	Total all students graduated

3. Name of teacher (not necessarily the same as the sponsor) employed for at least two semesters whose primary teaching field is mathematics and who has completed an undergraduate mathematics major or its equivalent.
   Shelby L. Harris

Major field of the teacher Mathematics Highest degree earned Doctor of Educati

in 1974 at University of Southern Mississippi
(Year) (Name of College or University)

List courses by title this teacher has taken above calculus: (If more than six courses are involved list only six)
Adv Calculus I, II, Matrix Algebra, Modern Alg I, II, Adv. Algebra,

4. Name of faculty sponsor Dr. Shelby L. Harris
5. Attached is a certificate order blank giving names of all full-member initiates. The Charter fee of $2.00, plus ~~$1.00~~ 2.00 initiation fee for each full member, totaling $ 52.00, is enclosed.

(Signature of Faculty Sponsor)

This petition is approved by the principal of the school.

Charles V. Wright, President
(Signature of Principal or Chief Administrative Officer)
10-26-81

ORDER BLANK

for

Membership Certificates

(Please type. Check spelling with care.)

Name of Chapter East Central Jr. College (school) Date of Order Feb 27,1982

School Address Box 41 ECJC (street or po box number)

Decatur. Mississippi 39327 (city, state, zip code)

Falucty Sponsor Dr. Shelby L. Harris

Date of initiation March 25 New members 10 Amount enclosed ($2.00/stu.)$ 20

First name Middle initial Last name Expected year of graduation

Billy M. Bounds 1983
Mattie Burnside 1983
Graham Clarke 1983
Gwyn A Guthrie 1983
Bernard Lee 1983
Lisa Miles 1983
Phil Smith 1983
Tammy Webb 1983
Gregory C Winste ad 1983
Deborah D.McGee 1983

We hereby certify that the above named students meet all the qualifications for full membership in Mu Alpha Theta and have been duly elected by this chapter.

Vikki Jenkins Chapter Secretary [illegible] Faculty Advisor

Mail to: Mu Alpha Theta, 601 Elm ave, Rm. 423, Norman, OK 73019

Order Blank for
Charter Member
Membership Certificates
(Please type. Check spelling and addresses with care)

Date of Order Oct 30. 1981

Name of Chapter East Central Jr. College, Decatur, Mississippi 39327
(School) (City) (State) (Zip Code)

Send certificates to: Dr. Shelby L. Harris, ECJC, Bx 41, Decatur, Ms 39327
(Name and Address)

Name of local newspaper Newton Record Newton Ms 39345 No. of initiates 25 Amount enclosed $50.00

First name | Middle initial | Last name | Expected year of graduation

1. Charlotte Chamblee 1983
2. Chris Clark 1983
3. Michelle Fulcher 1983
4. Donna Gibbs 1983
5. Roger Gunn 1982
6. Greg Henley 1983
7. Vikki Jenkins 1982
8. David Felton 1983
9. Bruce Lewis 1982
10. Dinah Moss 1983
11. Richard Nail 1983
12. Derek Pace 1983
13. Stella Posey 1983
14. Ramona Pullen 1983
15. Randy Russell 1982
16. Terri Russell 1982
17. Angèla Ryals 1983
18. Greg Shoemaker 1983
19. Ken Sims 1983
20. Bruce Sloan 1983
21. Becky Stanford 1983
22. Beth Tucker 1983
23. Robert Wade 1982
24. Ken Wallace 1983
25. Larry Germany 1983
26.
27.
28.
29.
30.
31.
32.
33.
34.
35.
36.
37.
38.
39.
40.

Please find check for $50.00 plus $2.00 Cash of Charter.

Date Charter November 1, 1981 if it does not make any difference.

I hereby certify that the above named students meet all of the qualifications for full membership in Mu Alpha Theta and have been declared Charter Members.

(Signature of Faculty Sponsor)

ORDER BLANK

for

Membership Certificates

(Please type. Check spelling with care.)

Name of Chapter East Central Junior College (school) Date of Order Nov. 11,'82

School Address Box 41 ECJC (street or po box number)

Decatur, Mississippi 39327 (city, state, zip code)

Falucty Sponsor Dr. Shelby L. Harris

Date of initiation December 9 New members 9 Amount enclosed ($2.00/stu.)$ 18.00

First name	Middle initial	Last name	Expected year of graduation

Wayne Bishop 1984
Tod Harrison 1983
Shawn Henry 1984

Karen Sue Hudspeth 1984
Ralph Hurley
Belynda Dawn Kemp 1984
Steve Mayes 1984
Kevin Smith 1984
John Gary Williams 1984

We hereby certify that the above named students meet all the qualifications for full membership in Mu Alpha Theta and have been duly elected by this chapter.

Ken Wallace
Chapter Secretary

Faculty Advisor

Mail to: Mu Alpha Theta, 601 Elm ave, Rm. 423, Norman, OK 73019

ORDER BLANK

for

Membership Certificates

(Please type. Check spelling with care.)

Name of Chapter East Central Junior College Date of Order Oct.3,1983
school

School Address ECJC Box 41
street or po box number

Decatur, Mississippi 39327
city state zip code

Falucty Sponsor Dr. Shelby L. Harris

Date of initiation Oct 20 New members 8 Amount enclosed ($2.00/stu.)$ 16.00

First name Middle initial Last name Expected year of graduation

David KARL Boydstun 1983

Joe ALAN Clay 1983

JERRY Don Cook 1983

Alan DALE Rhea 1983

Pam C. Savell 1983

Pam A. Thompson 1983

David L. Tidwell 1983

Pat G. Waldrip 1983

We hereby certify that the above named students meet all the qualifications for full membership in Mu Alpha Theta and have been duly elected by this chapter.

Chapter Secretary

Faculty Advisor

Mail to: Mu Alpha Theta, 601 Elm ave, Rm. 423, Norman, OK 73019

ORDER BLANK

for

Membership Certificates

(Please type. Check spelling with care.)

Name of Chapter: EAST CENTRAL JUNIOR COLLEGE (school) Date of Order: 3-11-83

School Address: Box 41, ecjc (street or po box number)

Decatur, Mississippi 39327 (city, state, zip code)

Falucty Sponsor: Dr. Shelby L. Harris

Date of initiation: April 7 New members: 11 Amount enclosed ($2.00/stu.)$ 2200

First name	Middle initial	Last name	Expected year of graduation

JAMES R. BROOKS 1984

DONALD GERMANY 1984

RODNEY GUNN 1984

GAYLE HARRELL 1984

ROBERT KILPATRICK 1984

MARY MATLOCK 1984

DARRYL MEADOR 1984

TIM TRIPLETT 1984

DAVID HERRING 1984

KAREN TINGLE 1983

MELESIA SPENCE 1984

We hereby certify that the above named students meet all the qualifications for full membership in Mu Alpha Theta and have been duly elected by this chapter.

Ken Wallace
Chapter Secretary

Faculty Advisor

Mail to: Mu Alpha Theta, 601 Elm ave, Rm. 423, Norman, OK 73019

PRESIDENT:

"I invite you to repeat after me the pledge which admits you into full membership."

PLEDGE:

"I, ______ do solemly promise to uphold the standards of Mu Alpha Theta and to make its purpose foremost in my mind, and I do solemly pledge allegiance to my fellow members and promise to aid them in search of mathematical truths."

"As your name is called, come forward and affix you name to the oath" ______________ will present each of you with your membership certificate."

SIGN CLUB ROSTER OF MEMBERS

SPONSOR:

"I now declare you members of Mu Alpha Theta:

It is my pleasure to welcome you into Mu Alpha Theta, an International Junior College Mathematics Society, and the lively fellowship of scholars it affords. I salute you for your accomplishments in mathematics: I charge you to explore always the mathematical truths, and dedicate yourselves to the cultivation of the well-rounded life, a prelude to service and honor in your community, state and nation."

Mu Alpha Theta
Initiation Ceremony

President:

"Candidates, you have presented yourselves for initiation in Mu Alpha Theta, an International Junior College Mathematics Society.

It is an honor to you to have been selected for membership in this international organization which has over 2000 chapters in all fifty states and in some foreign countries.

You each meet the requirements which include work done with distinction in college preparatory mathematics and in general high school work.

You have demonstrated ~~abili~~ty to work with others and to make personal research, and you possess qualities of industry, initiative, and reliability."

PURPOSE:

The Major Purpose of Mu Alpha Theta is to stimulate a deeper and more effective interest in mathematics.

Vice-President:

"Before you is a replica of the society's insignia. Its blue represents truth as unlimited as the sky. The Gold shines as a symbol that mathematics is, indeed, a valuable treasure. It represents, above all, a high point in the history of the world, a supreme law of mathematics.... one combining the mystery, the challenge, and the beauty of numbers with an indispensable law of geometry.

LET This symbol be a challenge to you - A symbol of encouragement.

MU ALPHA THETA

Order Blank

for

Membership Certificates

(Please type. Check spelling with care)

Name of Chapter/School East Central Jr. College Date of Order 11-15-8

School Address Box 41,ECJC
street of P. O. Box Number

Decatur, Mississippi 39327
city state zip code

Faculty Sponsor Dr. Shelby L. Harris

***Total Number of Members this Year 18+1 Date of Initiation 12- 6-85

New Members 18 Amount enclosed ($2.00 per student) $36.00 +$2.00

First Name Middle Initial Last Name Expected Year of graduation

HAROLD BLOCKER	1986
DAVID BRYANT	1986
MICHAEL CLOUD	1987
KERRY DAVIS	1987
KENNETH GOGGINS	1987
JOYCELYN GUNTER	1986
PATRICK NASH	1987
DALE PICKETT	1987
CARL PORTER	1987
PAT RIVES	1987
ALBEN ROLAND	1986
NEAL ROSAMOND	1987
CINDY SLOAN	1987
KEVIN THOMPSON	1987
ROBERT VALENTINE	1987
JOHNIE WEEMS	1987
KIM WILCHER	1987
GREG WOLVERTON	1987
DOUG WOODWARD	1987

We hereby certify that the above named students meet all the qualifications for full membership in Mu Alpha Theta and have been duly elected by this chapter.

Chapter Secretary Faculty Sponsor

Mail to: Mu Alpha Theta, 601 Elm Avenue, Room 423, Norman, OK 73019

Mu Alpha Theta

Order Blank for Membership Certificates

(Please type. Allow two weeks for processing)

Date of Order 4-20-98

Chapter ID Number 1687

Name of Chapter East Central Comm. College   School Phone 601-635-2111 ext. 238
Area Code & Number

P.O. Box 129   Decatur, MS   39327
Address   City   State   Zipcode +4

Faculty Sponsor Lois McMullan   Date of Initiation April 30

New Members 17   Amount Enclosed($3/student) #54   Total Members in All 95
#51 + #3 (See bill sent 3-25-98)

First Name	Middle Initial	Last Name	Year of Graduation

Adkins, Susan	1999
Adnesen, Silje	1999
Baskin, John	1999
Bryan, Josh	1999
Chesney, Lisa	1999
Duncan, Brandi	1999
Faulk, Laura Hope	1999
Griffin, Lorenzo	1999
Hollingsworth, Harley	1998
Morgan, Daniel	1999
Moss, Anita	1999
Sanders, Vanessa	1999
Sikes, Rebecca	1999
Stribling, Jodi	1999
Walker, Matt	1999
Willis, Rachel	1999
Young, Kim	1999

****** List Associate Members Separately ******

We hereby certify that the above named students meet all the qualifications for full membership in Mu Alpha Theta and have been duly elected by this chapter.

Ginger Greer
Chapter Secretary

Faculty Sponsor

Mail to : Mu Alpha Theta, 601 Elm Ave., Room 423, Norman, OK 73019-0315

MU ALPHA THETA INITIATION CEREMONY

PRESIDENT:

Candidates, you have presented yourselves for initiation in Mu Alpha Theta, an international honorary mathematics society. It is an honor to you to have been selected for membership in this society which has over 2000 chapters in all fifty states and in some foreign countries. You each meet the requirements which include work done with distinction in college mathematics. You have demonstrated that you possess the qualities of industry, initiative, and reliability.

VICE-PRESIDENT:

Before you is a replica of the society's insignia. Its blue represents truth as unlimited as the sky. The gold shines as a symbol that mathematics is a valuable treasure. The insignia represents, above all, a supreme law of mathematics...one combining the mystery, the challenge, and the beauty of numbers with an indispensable law of geometry.

SECRETARY:

Mu Alpha Theta was formed to engender keener interest in mathematics, to develop sound scholarship in the subject, and to promote enjoyment of mathematics among students.

PRESIDENT:

The society's name carries with it the meaning of the organization. **Mu** stands for the Greek word for learning; **Alpha** stands for the Greek word for truth; and **Theta** stands for the Greek word for service. I invite you now to stand and to repeat after me the pledge which admits you into full membership.

PLEDGE: "I, ____________________, do solemnly promise to uphold the standards of Mu Alpha Theta and to remain true to its purpose, and I do solemnly pledge allegiance to my fellow members and I promise to aid them in the search for mathematical truths."

You may be seated. As your name is called, please come forward, sign the membership roster, and receive your membership certificate.

SPONSOR: I now declare you members of Mu Alpha Theta. It is my pleasure to welcome you into our

MU ALPHA THETA INITIATION CEREMONY

PRESIDENT:

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PLEDGE: "I, ____________________, do solemnly promise to uphold the standards of Mu Alpha Theta and to remain true to its purpose, and I do solemnly pledge allegiance to my fellow members and I promise to aid them in the search for mathematical truths."

You may be seated. As your name is called, please come forward, sign the membership roster, and receive your membership certificate.

SPONSOR: I now declare you members of Mu Alpha Theta. It is my pleasure to welcome you into our

SPONSOR: I now declare you members of Mu Alpha Theta. It is my pleasure to welcome you into our organization and to commend you on your accomplishments in mathematics. I challenge you to always strive to do your best while you are here at ECCC and to dedicate yourself to cultivating a well-rounded life, a life of service to others, and a life of honor in your community.

Mathematics Club

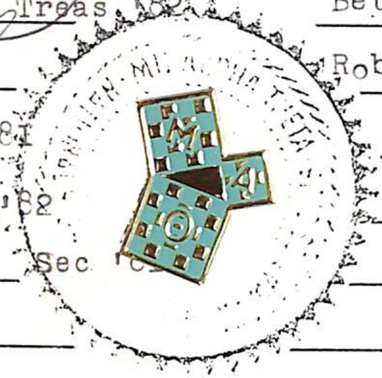
Mu Alpha Theta

Roster of Members

DATE INITIATED — **Charter Member** — **Junior College Division** — MEMBER

December 3,1981

Name	Signature	Office
Charlotte Chamblee	Charlotte Chamblee	
Chris Clark	Chris Clark	
David Felton	David Felton	
Michelle Fulcher	Michelle Fulcher	
Larry Germany	Larry Germany	Treas '82
Donna Gibbs	Donna Gibbs	
Roger Gunn	Roger Gunn	Pres '81
Greg Henley	Greg Henley	Pres '82
Vikki Jenkins	Vikki Jenkins	Sec '[illegible]
Bruce Lewis	Bruce Lewis	
Dinah Moss	Dinah Moss	
Richard Nail	Richard Nail	
Derek Pace	Derek Pace	
Stella Posey	Stella Posey	Social '81
Ramona Pullen	Ramona Pullen	Reporter '82
Randy Russell	Randy Russell	V Pres '81
Terri Russell	Terri Russell	Tr '81
Angela Ryals	Angie Ryals	



Name	Signature	Office
Greg Shoemaker	Greg Shoemaker	
Ken Sims	Ken Sims	VPres '82
Bruce Sloan	Bruce Sloan	
Becky Stanford	Becky Stanford	
Beth Tucker	Beth Tucker	
Robert Wade	Robert Wade	
Ken Wallace	Ken Wallace	Sec '82

Form 4

Mathematics Club

Mu Alpha Theta

Roster of Members

Junior College Division

DATE INITIATED	MEMBER	
March 25, 1982		
Billy Bounds	Bo Bounds	
Mattie Burnside	Mattie Burnside	
Graham Clarke	Graham Clarke	Social '82
Gwyn Guthrie	Gwyn Guthrie	
Bernard Lee	Bernard Lee	
Deborah McGee	Deborah McGee	
Lisa Miles	Lisa Miles	
Phil Smith	D Phil Smith	
Tammy Webb	Tammy Webb	
Greg Winstead	Greg Winstead	
December 9, 1982		
Wayne Bishop	Wayne Bishop	
Tod Harrison	Tod Harrison	
Shawn Henry	Shawn Henry	
Karen Sue Hudspeth	Karen Hudspeth	
Ralph Hurley	Ralph Hurley	
Belynda Dawn Kemp	Belynda Kemp	
Steve Mayes	Steve Mayes	

	MEMBER	
December 9, 1982		
Kevin Smith	Kevin Smith	
John Gary Williams	John Gary Williams	
March 24, 1983		
~~James Brooks~~	James Brooks	
Donald Germany	David Germany	
Rodney Gunn	Rodney Gunn	
~~Gayle~~ Harrell	Gayle Harrell	V. Pres.
David Herring	David Herring	
Robert Kilpatrick	Robert Kilpatrick	
Mary Matlock	Mary Matlock	
Darryl Meador	Darryl Meador	
Melesia Spence	Melesia Spence	Secretary
~~Karen Tingle~~		
55 ~~Tim Triplett~~	Timothy Triplett	Sec. 83

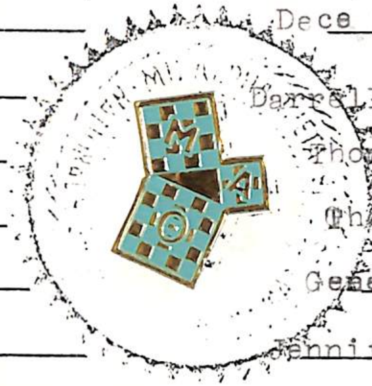
Form 4

Mathematics Club

Mu Alpha Theta

Roster of Members
Junior College Division

DATE INITIATED	MEMBER
October 13, 1983	
David Karl Boydstun	David Karl Boydstun
Joe Alan Clay	Joe Alan Clay
Jerry Don Cook	Jerry Don Cook
Alan Dale Rhea	Alan Dale Rhea
Pam C Savell	Pam Savell
Pam A Thompson	Pam Thompson
David L Tidwell	David L. Tidwell
Pat G. Waldrip	Pat G. Waldrip
February 17, 1984	
Johnny Bell	Johnny Bell
Amy Cox	Amy Cox
Charles Edward's	Charles Edwards
Alton Evans	Alton Evans
Chris Gilmer	Chris Gilmer V. Pres. 84
Deborah Graham	Deborah Graham
Rick Griffith	Ricky Griffith
Rhonda Hollingsworth	Rhonda Hollingsworth
Roger Lique	Roger Lique



	MEMBER
Dessa Mercer	Dessa Mercer Sec. 84
Stan Parkes	Stan Parkes
Marlin Savell	J Marlin Savell
Sarah Ware	Sarah Ware
Joe Williams	Joe Williams Secretary 84
December 6 1984	
Darrell Mangrum	Darrell Mangrum Treasurer 84
Thomas Bishop	Thomas Bishop
Thomas Booker	Thomas Booker
Gene Brumett	Gene Brumett, Jr.
Jennifer Caron	Jennifer Caron
Pam Eichelberger	Pamela Eichelberger
Phoebe Gray	Phoebe Gray
Fred Hamilton	[illegible]
Stan Hardy	Stan Hardy
Randy Hayes	Randy Hayes
Tammy Hollingsworth	Tammy Hollingsworth
Annette Hurley	Annette Hurley
Chad Kea	Chad Rea

Form 4

Mathematics Club

Mu Alpha Theta

Roster of Members
Junior College Division

DATE INITIATED	MEMBER
December 6, 1984(Continued)	
John T. Kelly	John T. Kelly
Brooks McElhenney	Brooks McElhenney
Greg McNair	Greg McNair
Marcia Mason	Marcia Mason
Eddie Pierce	Eddy Pierce
Annette Pillsbury	Annette Pillsbury
Patty Reeves	Patty Reeves
Molly Rushing	Molly Rushing
Shanan Scarborough	Shanan Scarborough
Terry Sessums	Terry Sessums
Felecia Taylor	Felicia Taylor
Tony Thrasher	Tony Thrasher
Cecilia Waggoner	Cecilia Waggoner
Greg Ware	Greg Ware
Charlotte Weems	Charlotte Weems
Sally Wilkerson	Sally Wilkerson
Doug Williams	Doug Williams
Patsy Williams	Patsy Williams

	MEMBER
DECEMBER 5, 1985	
HAROLD BLOCKER	Harold Blocker
DAVID BRYANT	William David Bryant
MICHAEL CLOUD	Michael Cloud
KERRY DAVIS	Kerry L. Davis
KENNETH GOGGINS	Kenneth Goggins
JOYCELYN GUNTER	Joycelynn Gunter
PATRICK NASH	Patrick J. Nash
DALE PICKETT	James Dale Pickett
CARL PORTER	Carl Porter
PAT RIVES	Patricia Rives
ALBEN ROLAND	Alben Roland
NEAL ROSAMOND	Neal Rosamond
CINDY SLOAN	Cindy Sloan
KEVIN THOMPSON	Thompson, Kevin C.
ROBERT VALENTINE	Robert Valentine
JOHNIE WEEMS	Johnie Weems
KIM WILCHER	Kim Wilcher
GREG WOLVERTON	Greg Wolverton
DOUG WOODWARD	Doug Woodward

Mathematics Club

Mu Alpha Theta

Roster of Members

Junior College Division

DATE INITIATED	MEMBER
April 10, 1986	
TAMMY COPELAND	Tammy Copeland
BARRY CRANE	Barry Crane
MARGUERITE DENNIS	Marguerite Dennis
GEORGE FISHER	George A Fisher
GREG LATHEM	Greg Lathem
CARL LOCKHART	Carl Lockhart
SCOTT MOORE	Charles S. Moore
MOLLY RUSHING	Molly Rushing
RICHARD SMITH	Richard Smith
APRIL WATTS	Aprille Watts
JENNIFER YOUNG	
December 4, 1986	
Julie Adams	Julie Adams
Vicki Arthur	Vicki Arthur
Brenda Dearing	Brenda Dearing
Danelle Eversoll	Danelle Eversoll
Shelia Goodwin	Sheila Goodwin
Mary Ann Gray	Mary Ann Gray

	MEMBER
Thurman Harrell	Thurman Harrell
Seleta Howard	Seleta Howard
Melanie Huffman	Melanie Huffman
Meschelle Jacobs	Meschelle Jacobs
Suzanna Kennedy	Suzanna Kennedy
Terry Matula	
Jeff Neal	Jeff Neal
Phillip Nelson	Phillip Nelson
John Stokes	John Stokes
Marsha Stovall	Marsha Stovall
Jason Thomas	Jason Thomas
Tim Thompson	
Melissa Thrash	Melissa Thrash
Marshall Watkins	Marshall Watkins
Shelley Wolverton	Shelley Wolverton

Form 4

Mathematics Club

Mu Alpha Theta

Roster of Members
Junior College Division

DATE INITIATED	MEMBER
December 8, 1987	
Mark Amis	Mark Amis
Leanne Brantley	Leanne Brantley
Chris Bunty	Chris Bunty
Jamie Davidson	Jamie Davidson
Steve Dean	Steven Dean
Carol Edwards	Carol Edwards
Tina Eubanks	Tina Eubanks
Bill Freeman	Bill Freeman
Tanya Henry	Tanya Henry
Ryan Hull	Ryan Hull
Sedera Irons	Sedera Irons
Maggie Jennings	Maggie Jennings
Robert Kennedy	Robert Kennedy
Tammy Kirkland	Tammy Kirkland
Josef Mainka	Josef Mainka
Brian McDonald	Brian McDonald
Michelle Mitchell	Michelle Mitchell
Reggie Shumaker	Reggie Shumaker

	MEMBER
Marc Smith	Marc Smith
Rhonda Smith	Rhonda Smith
Teresa Stroud	Teresa Stroud
Tammie Todd	Tammie Todd
Tina Walker	Tina Walker
Heather Watts	Heather Watts
Missy Whitmire	Missy Whitmire
Debora Wilson	Debora Wilson
Scott Wade	Wade Scott
Daryle Brown	Daryl Brown
Leigh Ann Russell	Leigh Ann Russell

Form 4

Mathematics Club

Mu Alpha Theta

Roster of Members
Junior College Division

DATE INITIATED	MEMBER		MEMBER
Chris Coker			
Mike Cooper	Mike Cooper		
Ruth Cran	Ruth Cran		
Kay Germany	Kay Germany		
Rhonda Loper	Rhonda Loper		
Karen Mayo	Karen Mayo		
Stanley McDill			
Ronald McGilbra	Ron McGilbra		
Nikki Minard	Nikki Minard		
Nina Mosley	Nina Moseley		
Tammy Shirley			
Tammy Sullivan	Tammy Sullivan		

Form 4

Mathematics Club

Mu Alpha Theta

Roster of Members
Junior College Division

DATE INITIATED	MEMBER
Dec. 8, 1988	
Alicia Amis	
Alan Anthony	Alan Anthony
Mike Arthur	Mike Arthur
Dana Bates	Dana Bates
Candi Beckham	Candi Beckham
Paula Carter	
Amy Cockroft	
Bodie Copeland	Bodie Copeland
Stephanie Gainey	Stephanie Gainey
Cindy Hall	Cindy Hall
Reed Kilpatrick	Reed Kilpatrick
Bruce Mainka	Bruce Mainka
James Mooneyham	James E Mooneyham
Connie Myers	Connie Myers
Angell Posey	Angell Posey
Lisa Risher	Lisa Risher
Stan Smith	Stan Smith
Dexter Thornton	Dexter Thornton

	MEMBER
Todd Thornton	Todd Thornton
Heather Weidler	Heather Weidler
Brent Wells	Brent R. Wells
Kerry Wilcher	Kerry Wilcher
May 4, 1989	
Johnny Beaver	Johnny Beaver
Darlene Butler	Darlene Butler
Lee Chamblee	Lee Chamblee
Ben Guthrie	Ben Guthrie
Melissa Hatcher	Melissa Hatcher
Stephanie Patrick	Stephanie Patrick
Duane Richardson	Duane Richardson
Sadie Triplett	Sadie Triplett
Rachel Walls	Rachel Walls

Form 4

Mathematics Club

Mu Alpha Theta

Roster of Members
Junior College Division

DATE INITIATED	MEMBER		...ER
12-6-90		KARLA MUNN	
ANGELA ANDERSON		ANGELA NICHOLSON	
LAMAR BLALOCK	Lamar Blalock	DARLENE PATE	Darlene Pate
CHAD BRELAND		MATT STRUM	Matthew Strum
RUSH CALLAHAN	Rush Callahan	MICHELE WADE	Michele Wade
KIM COCKRELL		JASON SMITH	Jason Smith
SHELIA COOK	Shelia Cook	TRACY WHITE	
TAM COOPER	Tam Cooper	CHASITY YOUNG	Chasity Young
MACHELLE DUNN		RICKY COOK	Ricky Cook
CINDY FERGUSON	Cindy Ferguson	BRIAN JONES	
NICOLE FLINT		DEVLIN HUFFMAN	
KIM GRIFFIN	Kim Griff	RICHIE RIVERS	Richie Rivers
AB GERMANY			
ALEX HOYE	Alex Hoye		
BOBBY JENKINS			
CATHIE JONES	Cathie Jones		
SHARMON JONES			
LELA LADNER	Lela Ladner		
JEANIA LITTLE	Jania Little		

Form 4

Mathematics Club

Mu Alpha Theta

Roster of Members

Junior College Division

DATE INITIATED	MEMBER		MEMBER
12- 6-90			
Angela Anderson		Lela Ladner	
Lamar Blalock	Lamar Blalock	Jeania Little	
Chad Breland	Chad Breland	Karla Munn	Karla Munn
Rush Callahan		Angela Nicholson	Angela Nicholson
Ricky Cook	Ricky Cook	Darlene Pate	
Shelia Cook		Richie Rivers	
Tam Cooper	Tam Cooper	Jason Smith	~~Matt~~
Machelle Dunn	Machelle Dunn	Matt Strum	Matthew Strum
Cindy Ferguson		Michele Wade	
Nicole Flint		Tracy White	
Ab Germany	Ab Germany	Chasity Young	
Kim Griffin	Kim Griffin		
Alex Hoye	Alex Hoye		
Devlin Huffman	Devlin Huffman		
Bobby Jenkins	Bobby Jenkins		
Brian Jones			
Cathie Jones			
Sharmon Jones	Sharmon Jones		

MU ALPHA THETA

Form 4

Mathematics Club

Mu Alpha Theta

Roster of Members
Junior College Division

DATE INITIATED	MEMBER		MEMBER
May 3, 1990	LAFAYETTE CARTER		Lafayette Carter
	DONNA EDWARDS		Donna Edwards
	SCOTT FULCHER		Bridgette Futch
	BRIDGETTE FUTCH		Mitchell Graham
	MITCHELL GRAHAM		Dana Gressett
	DANA GRESSETT		
	TERRISA HANCOCK		Terisa B. Hancock
	MELISSA HILLMAN		
	GREG JACKSON		
	BILLY JOHN		Billy John
	JANIS JOHNSON		Janice Johnson
	TRACY LADD		
	SONYA LEWIS		Sonya Lewis
	SHEA OAKLEY		Shea Oakley
	DAVID PITTS		
	DAVID SHAW		David Shaw
	DWAYNE SMITH		Dwayne Smith
	KAY VAUGHN		Kay Vaughn
	BECKY WHEELER		Becky Wheeler

Form 4

Mathematics Club

Mu Alpha Theta

Roster of Members
Junior College Division

DATE INITIATED	MEMBER
May 7, 1991	
Tracy Adcock	Tracy Adcock
Allen Boutwell	Allen Boutwell
Rene Breedlove	Rene Breedlove
Eric Butler	Eric Butler
Jamey Champion	Jamey Champion
Lynn French	Lynn French
Patsy Harris	Patsy Harris
Chad Harrison	Chad Harrison
Melissa Hillman	Melissa Hillman
Jay Kerr	Jay Kerr
Andy Magoun	Andy Magoun
Eddie Rutherford	Eddie Rutherford
Polly Spivey	
DECEMBER 5, 1991	
Jennifer Anderson	Jennifer Anderson
Kimberly Boman	Kimberley Boman
Tracey Blackburn	Tracy Blackburn
Angela Boutwell	

	MEMBER
Stacy Copeland	Stacy Copeland
Misty Cornett	
Lynnetta Cooksey	Lynnetta Cooksey
Tonya Durant	
Stephen Ford	Stephen Ford
Jay Fortenberry	Jay Fortenberry
Bruce Lee	Bruce Lee
Monica Logan	
Tammy Martin	
Jay Mills	
Brandi Parks	Brandi Parkes
Robbie Pierce	Robbie Pierce
Sonja Quinn	
Stephanie Rigdon	
Lori Rogers	Lori Rogers
Rochelle Sessums	
Adnon Shahid	Adnon Shahid
Noel Sharp	Noel Sharp
Stacey Smith	Stacey Smith

Form 4

Mathematics Club

Mu Alpha Theta

Roster of Members
Junior College Division

DATE INITIATED	MEMBER		
December 5, 1991 (continued)			
Johnny Thomas			
Kim Thornton	Kim Thornton		
Kasey Townsend			
Craig Youngblood	Craig Youngblood		
	Sonya Quinn		

Form 4

Mathematics Club

Mu Alpha Theta

Roster of Members

Junior College Division

DATE INITIATED	MEMBER
12/3/92	
DIANE ADKINS	Diane Adkins
DEIRDRE AMIS	Deirdre Amis
SCOTT BATTLE	Scott Battle
STACY BROOKS	
CINDY CALLAHAN	Cindy Callahan
AUDREY CLARK	
MARK FANNING	Mark Fanning
JUSTIN FISHER	P. Justin Fisher
ROCHELLE GADDIS	Rochelle Gaddis
TIMMY GENTRY	Timmy Gentry
JIMMY GIBBS	Jimmy Gibbs
MARSHA GRAHAM	Marsha Graham
BRANDON GRAY	Brandon Gray
JOHN HERRINGTON	John Herrington
CARLA LEFLORE	Carla Leflore
KAREN LONG	Karen Long
YANCY MASON	
SHELIA MCDILL	Shelia McDill

	MEMBER
LEANN MCMULLAN	
DANA POPE	
KIM REEL	Kim Reel
JENNIFER RICHMOND	
BEN RUDOLPH	
EDWARD SANDERS	Edward Sanders
LEIGH ANN SIGREST	Leigh Ann Sigrest
MELANDY THOMAS	Melandy Thomas
ROBBY THORNTON	
BILLY TURNER	
TERRY WOODSON	Terry Woodson

Form 4

Mathematics Club

Mu Alpha Theta

Roster of Members
Junior College Division

DATE INITIATED	MEMBER	MEMBER
May 4, 1993		John Ferguson
Steve Burton		Robin Stenden
Chris Croft		Hartman Holleman
Sharon Eldridge	Sharon Eldridge	Mike Hunt
Tammy Ferguson	Tammy Ferguson	Bridgett Joiner
Felicia Howard	Felecia Howard	Jenny Killen
John Hutto	John Hutto	Paul Killen
Tarsha Lagrone	Tarsha Lagrone	Christie Lowe
Jessica Myers	Jessica Myers	[illegible]
Amy Pace	Amy Pace	Michele McAdory
Wendy Phillips	Wendy Phillips	Tony Perozo
Rebecca Savell		Tiffany Phillips
Timothy Thompson	[illegible]	Kris Ligon
Derrick Tucker	Derrick P Tucker	Mandy Jessum
Fall 1993	Arlene Bates	Jenny Shows
	Robin Bush	Matt Smith
	Gwen Burton	Jonathan Spears
	Jacqueline Cain	Guy B M[illegible]
	RexAnne Chaney	Leslie Dalton

Form 4

Mathematics Club

Mu Alpha Theta

Roster of Members
Junior College Division

DATE INITIATED	MEMBER		[illegible]ER
May 3, 1994	Sharon K. Blaine		
May 3, 1994	Letizica Bobbitt		
May 3, 1994	Anna M. Smith		

Form 4

Mathematics Club

Mu Alpha Theta

Roster of Members
Junior College Division

DATE INITIATED	MEMBER
5-2-95	Amy Phillips
5-2-95	Shane [illegible]
5-2-95	Corey Alford
	[illegible]
5-2-95	Shannon Goss
5-2-95	Brad Garrett
5-2-95	Cherie McElhenney
5-2-95	Phyllis Morrison
5-2-95	Melanie Carson
5-2-25	Florida Lorders

DATE INITIATED	MEMBER

Form 4

Mathematics Club

Mu Alpha Theta

Roster of Members
Junior College Division

DATE INITIATED	MEMBER		MEMBER
1-19-95		1-19-95	
Randy Bain		Rebecca Nickey	
Stephanie Baskin	Stephanie Baskin	Ty Nowell	
Jennifer Bonner	Jennifer Bonner	Jody Riser	
Valeria Buggs	Valeria Buggs	Alex Savell	
Tim Chapman		Matt Skinner	Matt Skinner
Melanie Cleveland		Ryan Smith	Ryan Smith
Dawn Cockrell	Dawn Cockrell		
Heather Dittmer	Heather Dittmer		
Katherine Faulkner	Kat Faulkner		
Pam Ferguson	Pam Ferguson	Staci Strebeck	
Kyra Fulton	Kyra Fulton	Craig Vowell	Craig Vowell
Mandy Goldman	Mandy Goldman	Renee Wheeler	
Melissa Jones	Melissa Jones	Jana Wilkerson	Jana Wilkerson
Jill Kirkland	Jill Kirkland	Michael Wilson	Michael Wilson
Elizabeth Knochenmuss	Elizabeth Knochenmuss	Dan Young	
Brandi Laird	Brandi Laird		
Melissa McDill	Melissa McDill		
Kelly Miller	Kelly Miller		

Mathematics Club

Mu Alpha Theta

Roster of Members
Junior College Division

DATE INITIATED	MEMBER		MEMBER
12-7-95	Amy Boatner		
12-7-95	Teri Cooks		
12-7-95	Olivia Hamil		
12-7-95	Karen McRae		
12-7-95	Jonathan Sharp		
12-7-95	Andy Sistrunk		
12-7-95	Tonya Williamson		
12-7-95	Angela Anthony		
12-9-95	Adella Rainer		
12-7-95	Amy Monk		
12-7-95	Kathy Easterling		
04-30-96	Lise Beeland		
4/30/96	Bart Dugan		
4 30 96	Justin G[illegible]		
4-30-96	Karen Parsons		
4-30-96	Kathy Whitt		
4-30-96	Amy Moore		

Form 4

Mathematics Club

Mu Alpha Theta

Roster of Members
Junior College Division

DATE INITIATED	MEMBER		MEMBER
12-5-96	Madonna Atkinson	12-5-96	LaRita Williams
12-5-96	Deanna Bankston	12-5-96	Angela King
12-5-96	Amanda Bell	12-5-96	Chris Stewart
12-5-96	Amy Dunaway	12-5-96	Nathan King
12-5-96	Ricky Frazier	12-5-96	Detra Carter
12-5-96	Amanda Gilbert	4/29/97	Donnie C. Atkinson
12/5/96	Amy Gomillion	4/29/97	Allyson McPhail
12-5-96	Ginger Green	4-29-97	Yancie Miles
12-5-96	Sissy Green	4/29/97	Sashoney Nelson
12/5/96	Janet Harrison	4/29/97	Brian Sha[illegible]
12/5/96	Jacob M. Jenkins	4/29/97	Lesley Weems
12/5/96	Jennifer McDaniel		
12/5/96	Justin Miles		
12/5/96	Tonya Reed		
12/5/96	Krisan Risher		
12/5/96	Kim Shaffer		
12/5/96	Amy Spears		
12-5-96	Adria Stamper		
12-5-96	Leslie Swindle		

Form 4

Mathematics Club

Mu Alpha Theta

Roster of Members
Junior College Division

DATE INITIATED	MEMBER	DATE INITIATED	MEMBER
	5-4-99		12-2-99
	Jennifer Bounds		Leanne Earls
	Francis Brown		Jessica Evans
	Stacy Cotten		Tiffany Dahood
	Ginny Rigby		Kelli Harris
	Michael Shumaker		Amy Hatchey
	Laura Shows		Kristin Johnson
	Connie Jew		Sarah Johnson
	Kimberly Thomas		Cantrell Langley
	Kelly Tucker		Bridget Minor
	Marci LaSall		Jason Myers
	April Wallace		Lance Noel
	Emily Boggan		Edward H. Porter, Jr.
	Natalie Boggan		Jackie Porter
	Corey Bounds		Amanda Sessions
	Ramona Cakern		Kyle Watson
	Hal Davis		Michael Wilcox
	Ben Dear		
	Ashley Deel		

Form 4

Mathematics Club

Mu Alpha Theta

Roster of Members
Junior College Division

DATE INITIATED	MEMBER	DATE INITIATED	MEMBER
4-19-01	Jacqueline Moore	4-19-01	Brandi Bounds
4-19-01	Wesley [illegible]	3-01	Jessie Warren
4-19-01	[illegible]		
4-19-01	Cassandra Arrington		
4-19-01	Lena Ford		
4-19-01	Jessica Sanders		
4-19-01	Laura Williams		
4-19-01	Rebecca Lynn Kelley		
4-19-01	Lesley Taylor		
4-19-01	Matthew Stone		
3-01	Katy McBeath		
3-01	Allison Latham		
3-01	James B. Parker		
3-01	Samantha Styburski		
4/19/01	Penny Alford		
4/19/01	Beth Bell		
4-19-01	Daniel Peace		
4-19-01	Matt Davis		
4-19-01	Jason Gurgedy		

Form 4

Mathematics Club

Mu Alpha Theta

Roster of Members

Junior College Division

DATE INITIATED	MEMBER		
3/28/02	Angela Holdiness	12-05-02	Angela E. Holmes
3/28/02	Mitzi K. Rogers	12-05-02	Susan Holland
8/28/02	Maranda Burks	12-05-02	Amy Rawson
3-28-02	Lynda W. Killen		Natasha Phillips
3-28-02	Beth Hollingsworth		Shelley Johnson
3-28-02	Amanda Lathem		Jennifer Harpole
3-28-02	Miranda Welch		Lacey Sanborn
3-28-02	Bryant Steele		Mandy Eakes
3-28-02	Samantha Gray		Christy Fedrick
3-28-02	Joseph Wall		Christy Martin
3-28-02	Chris Burla		David Burt
3-28-02	Heather Brantley		Kery McDill
03-28-02	Kathy S. Jones		Amy Warner
3-28-02	Kerri Burks		Amanda Loyd
3-26-02	[illegible]		[illegible]
3-28-02	Jaclyn B. Evans		Joseph Charlton
		12-5-02	Richard DeBow
			Dawn Cotten

Form 4

Mathematics Club

Mu Alpha Theta

Roster of Members
Junior College Division

DATE INITIATED	MEMBER		
4/24/2003	ashley Brown	10-20-05	Jaimee Jones
	Timothy Fuller		Cecily Zaffart
	aparna Goswami		Cory Shuddleford
	Vimukti Goswami		Tiffany S. Mills
	Brandi Brown		Cynthia Gilson
	Dan Henry		Melinda Walker
	Jennifer Kersh		Scott Death
	Robin R Page		
	Eric Townsend		
4-29-04	Ashley Clark		
12/2/04	Anna Price		
	Kyle Gordon		
	Lee Anne Rawson		
	Kimberly Coley		
	Amy Earley		
	Anthony Stafford		

Form 4

Mathematics Club

Mu Alpha Theta

Roster of Members
Junior College Division

~~DATE INITIATED~~ graduation year	MEMBER		
2009	Caiti Slay		Anna Johnson
2009	Santigous Culpepper		
2009	Rebecca Watson		
2008	Cundell Hickam		
2009	Brandon Williams		
2008	Cristen S. Jones		
2008	Lynette Bell		
2008	James D. Smith James D Smith		
2007	Brent A. Barton		
2009	Timothy L. [illegible]		
2008	Jennifer Distunk		
2008	Amneris Avalos		
2008	Jonathan Jackson		
2008	Joseph McDonald		
2009	Celeste Franklin		

Form 4

Mathematics Club

Mu Alpha Theta

Roster of Members
Junior College Division

DATE INITIATED	MEMBER		
2/19/09	Kurt Simoneau (2010)		
2/19/09	Bill Flowers (2010)		
2/19/09	Bobby Eaton (2010)		
	Brandi Gray (2009)		
	Hannah Watkins (2010)		
	meaghan mayes (2010)		
	Anna White (2009)		
	Anna Switzer (2010)		
	PATRICK PATTERSON (2009)		
	Casey Stokes (2009)		
	Michael Sanford (2009)		
	Fred Harris (2010)		
	Daniel Bankston (2010)		

Form 4

Mathematics Club

Mu Alpha Theta

Roster of Members
Junior College Division

DATE INITIATED	MEMBER		MEMBER
	Paul Sewell (2010)		
	Chelsea Pugh (2010)		
	Jason Watkins 2010		
	Celia Boggan (2011)		
	Reaghan Mayes (2010)		

Form 4

PURPOSE OF THE ORGANIZATION

Mu Alpha Theta was formed to engender keener interest in mathematics, to develop sound scholarship in the subject and promote enjoyment of mathematics among junior college students.

Meaning of Mu Alpha Theta

Mathema - that which is learned (mathematics comes from Mathema)

Mathematicians - from the Greek word Math e ma ti kos
meaning one disposed to learn

Mu - Math e sis - Learning

Alpha - Al e th eia - Truth

Theta - Ther a peia - Service

2 VICE*PRESIDENT

Before you is a replica of the society's insignia. Its blue represents truth as unlimited as the sky. The gold shines as a symbol that mathematics is, indeed, a valuable treasure. It represents, above all, a high point in the history of the world, a supreme law of mathematics.... one combining the mystery, the challenge, and the beauty of numbers with an indispensable law of geometry.

LET THIS SYMBOL BE A CHALLENGE TO YOU ,- A symbol of encouragement.

1. PRESIDENT:

Candidates, you have presented yourselves for initiation in Mu Alpha Theta, International junior college mathematics Society.

It is an honor to you to have been selected for membership in this international organization which has over 20000 chapters in all 50 states and in some foreign countries. You each meet the requirements which include work done with distinction in college preparatory mathematics and in general high shcool work, you have demonstrated ability to work with others and to make personal research, and you possess qualities of industry, initiative, and reliability.

The major PURPOSE OF MU ALPHA THETA is to stimulate deeper and more effective interest in mathematics.

VICE-President Speaks Here..... then President "

It is desirable that you be given instruction in the deeper meanings and historical importance of mathematics.

All your years.

After Larry

5 Secretary:

You have studied the constitution of Mu Alpha Theta, the prupose of which is to stimulate deeper and more effective interest in mathematics and whose principles are embodied in the Greek letters which stand for (Learning, Truth & Service)

Now that the standards and ideals of this organization have been fully revealed to you, you come to renew and complete the pledge which admits you into complete fellowship.

President:

It is desirable that you be given instruction in the deeper meanings and historical importance of mathematics.

SPEAKER I

SPEAKER II

SPEAKER III

PRESIDENT

The Challenge:

You are now instructed briefly in the history, the growth, the power, and the future of mathematics. More is needed of you. You must exchange freely your ideas, and you must continue to grow mathematically through all your years.

SECRETARY:

You have studied the constitution of Mu Alpha Theta, the purpose of which is to stimulate a deeper and more effective interest in mathematics and whose principles are embodied in the Greek letters M A Θ, which stand for (mathesis, aletheia, therapeia) - learning, truth, service.
Now that the standards and ideals of this organization have been fully revealed to you, you come to renew and complete the pledge which admits you into complete fellowship.

SPONSOR:
Candidates, arise. If you agree to accept this challenge, your answer should be 'I Do!'

CANDIDATES:

"I Do!"

Chapter Sponsor:

" it is my pleasure to welcome you into Mu Alpha Theta, International Junior College Mathematics Society, and the lively fellowship of scholars it affords. I salute you for your accomplishments in mathematics; I charge you to explore always the mathematical truths, and dedicate yourselves to the cultivation of the well-rounded life, a prelude to service and honor in your community, state and nation."

It is desirable that you be given instruction in the deeper meanings and historical importance of mathematics.

3 Purpose

PRESIDENT:

4 CHALLENGE:

You are now instructed briefly in the history, the growth, the powers, and the future of mathematics. More is needed of you. You must exchange freely your ideas, you must continue to grow mathematically through all your years.

SPONSOR:

5 Candidates, arise. If you agree to accept th is challenge, YOUR ANSWER SHOULD BE, "I DO".

CHAPTER PRESIDENT:

6 " I invite you to repeat after me the pledge which admits you into full membership."

" I ____________ do solemly promise to uphold the standards of Mu Alpha Theta and to make its purpose foremost in my mind, and I do solemmy pledge allegiance to my fellow members and promise to aid them in search of mathematical truths."

As your name is called, come forward and affix your name to the oath" _________________ will present each of you with your membership certificate."

SPONSOR:

I now declare you members of Mu Alpha Theta.

INSTALLATION OF LOCAL OFFICERS:

The officers to be installed shall stand in a semi-circle. Face the installing officer:

The pledge may be administered individually or in unison.

INSTALLING OFFICER SAYS:

"Before you assume your obligations as officers of Mu Alpha Theta, you shall pledge for membership in Mu Alpha Theta. Repeat after me"

" I do dolemnly promise to uphold the standards of Mu Alpha Theta, and to keep foremost in mind the purpose of the society.

INSTALLING OFFICER SAYS:

" you are charged with the responsibility of learning the duties of your office and of executing those duties to the best of your ability. Now repeat after me"

"I solemnly promise to fulfill to the best of my ability my duties as an officer of Mu Alpha Theta. "

INSTALLING OFFICER SAYS:

"You are duly recognized as the executive officers of Mu Alpha Theta, East Central Junior College Chapter. We pledge the support of the members of this chapter to you.

HANDBOOK

FOR

SPONSORS

MU ALPHA THETA

NATIONAL HIGH SCHOOL AND JUNIOR COLLEGE
MATHEMATICAL CLUB

NATIONAL HIGH SCHOOL AND JUNIOR COLLEGE

Mathematics Club

Mu Alpha Theta (M A Th)

This is your invitation to establish a chapter of The National High School and Junior College Mathematics Club, M A Th, in your school. The club, which is cosponsored by the Mathematical Association of America, and the National Council of Teachers of Mathematics is a non secret organization whose purpose is to stimulate interest in mathematics by providing public recognition of superior mathematical scholarship, and by promoting various mathematical activities. Its journal, the Mathematical Log, is constantly being praised as an outstanding medium for up-to-date, understandable articles about Mathematics. There are now over 1200 chapters in 46 states and in Canada, Japan, the Canal Zone, and Switzerland. If your school can qualify you owe it to your students to establish a chapter and to participate in their many stimulating activities.

I. Purpose of the Organization

The National High School and Junior College Mathematics Club, Mu Alpha Theta was formed to engender keener interest in mathematics, to develop sound scholarship in the subject and promote enjoyment of mathematics among high school and junior college students.

National policy is determined by a nine-person governing council consisting of three national officers (president, president-elect or immediate past president, and secretary-treasurer), four governors elected for three-year terms, and representatives appointed by the Mathematical Association of America and the National Council of Teachers of Mathematics. These organizations each have the privilege of nominating two of the seven elected council members. Others are nominated by a committee of the club. Officers and governors serve without remuneration.

The governing council provides the following services for chapters:

1. Individual Membership Certificates, billfold cards and School Charters are issued by the national Secretary-Treasurer.
2. The Mathematical Log, official journal of the organization, is distributed free to the chapters. It is devoted to understandable articles on mathematics, both modern and ancient, which have been especially written for the intended audience. The Log also contains current chapter news, news of regional and national meetings, suggestions of topics suitable for discussion at chapter meetings, discussion of recent books, periodicals, films, tape recordings, mathematical devices and similar items of interest to the membership.
3. One copy of our other publications, now six in number, are sent to each chapter without charge. Additional copies are available at a 40% discount.
4. Up-to-date information about mathematical careers is supplied to each chapter.
5. Local Newspapers are notified that a charter has been granted, and of the names of all students elected as full members.
6. Insignia pins and buttons are available from the National Office.
7. Regional meetings and other activities are constantly being arranged by the chapters.
8. National meetings are held with lectures by outstanding mathematicians, as well as by students.
9. Chapters are encouraged to make suggestions to the national officers concerning additional projects which they would like to see carried out. A sincere effort is made to be as helpful as possible.
10. The Handbook for Sponsors.

II. School Qualifications for Chapters

Any high school, two-year junior college, or other academic institution giving training equivalent to one of these, may petition to have a chapter providng it meets the following minimum requirements:

1. At least six semesters of mathematics including algebra, geometry and more advanced topics (or, in the case of junior colleges, courses including the calculus) must be offered. These requirements can not be fulfilled by courses in general mathematics, business mathematics, shop mathematics, or arithmetic.

2. During the two semesters preceding that in which a petition is submitted, the school must have employed at least one teacher whose primary teaching field is mathematics and who has completed an undergraduate mathematics major or its equivalent at an accredited college or university.

3. The Principal, or other chief administrative officer of the institution, must approve the petition.

4. An initial charter fee of $2.00, along with the regular initiation fee of $1.00 for each member, must accompany the petition for a charter. (These fees will be returned if the governing council votes that the institution is ineligible for membership.)

5. A favorable vote of two-thirds of the governing council shall be required to elect a chapter to membership. Each petitioning institution will be notified as soon as possible whether or not a charter has been granted. This takes about 3 weeks.

6. The petition should be sumbitted (along with 10 copies) on the enclosed form provided by the national office. The copies may be on plain paper.

III. Qualifications for Individual Membership

The following minimum requirements for full and associate membership shall be common to all chapters. Each chapter shall have a faculty-student committee which will recommend possible members for the chapter's consideration. No student shall be recommended for consideration who does not meet the minimum qualifications. Additional requirements may be imposed by individual chapters.

(a) Full Membership: High school students who have completed the equivalent of four semesters of college preparatory mathematics and in addition have completed or are enrolled in a still more advanced course, are eligible for full membership providing their mathematical work was done with distinction (on the ABCDF grading scale this shall mean at least a B average).

Junior College students are not eligible on the basis of their high school records alone: they must have in addition a B average in the equivalent of at least one semester of college mathematics which has as a prerequisite the equivalent of five semesters of college preparatory mathematics. Junior College members, however, who already were elected in high school continue to be members and are eligible to belong to a junior college chapter.

(b) Associate Membership: Persons who have completed two semesters of algebra or their equivalent with distinction and who have enrolled in a third semester of mathematics are eligible for associate membership. Associate members do not pay the $1.00 initiation fee and are not registered with national headquarters. They are not entitled to vote on national policy. They are entitled to attend and be heard at meetings, and presumably are likely candidates for full membership.

IV. National Finances

There shall be no national annual dues. An initiation fee of $1.00 per full member shall be paid to the national secretary-treasurer for each person initiated into full membership, where-upon the secretary-treasurer shall issue a membership certificate to that member and place his name on the official roll. A $2.00 charter fee shall be charged each new chapter at the time the official charter is first issued to the school.

Finances are supervised by the Controller's Office of The University of Oklahoma, Regular state supervised auditing service is provided, in addition to approved monthly machine accounting. Council approval is requied for expenditures other than printing and metered postage, and the secretary-treasurer must make complete accounting to the Council.

V. Local Organizations, Officers, and Finances

Each chapter is free to set up its own organization, officers, and finances with the following restrictions:

1. Each chapter must have a semi-permanent faculty-sponsor and the national office (secretary-treasurer) must be kept informed of the current faculty sponsor's name and address.

2. The minimum membership requirements set forth by the national office must be met by all initiates.

3. A complete and accurate list of all full member initiates, accompanied by their $1.00 initiation fees, must be received by the office of the secretary-treasurer before the initiation takes place.

4. Each chapter must hold regular meetings at periodic intervals and not merely consider itself an honor society for high grades. At the minimum, there should be one meeting every month.

5. Local chapters are encouraged to participate actively in the life of the school, providing stimulation of an interest in, and appreciation of, mathematics for all students.

6. Either the name National High School and Junior College Mathematics Club or Mu Alpha Theta may be used as desired.

VI. Governing Council

PRESIDENT: Ms. Katherine P. Layton, Beverly Hills High School, Beverly Hills, CA 90212
PAST PRESIDENT: Dr. Robert Kalin, Florida State University, Tallahassee, FL 32306
SECRETARY-TREASURER: Dr. Harold V. Huneke, University of Oklahoma, Norman, OK 73019
OFFICIAL REPRESENTATIVE of the MATHEMATICAL ASSOCIATION of AMERICA:
Dr. Robert Wilson, Ohio Wesleyan University, Delaware, OH 43015
OFFICIAL REPRESENTATIVE of the NATIONAL COUNCIL of TEACHERS of MATHEMATICS:
Mr. Alvin Gloor, Westside High School, Omaha, NB 68144

GOVERNORS:

Region I Mr. James Woolum, Clayton Valley High School, Concord, CA 94521
Region II Ms. Kathryn Hinsenbrock, Community High School, Charles City, IA 50616
Region III Ms. Adele Hanson, Milwaukee Technical High School, Milwaukee, WI 53204
Region IV Mr. Thomas Thrasher, Austin High School, Decatur, AL 35601

National High School and Junior College Mathematics Club
Mu Alpha Theta (M A TH)

601 Elm Avenue, Room 423, Norman, Oklahoma 73019

Cosponsored by The Mathematical Association of America and The National Council of Teachers of Mathematics

November 25, 1981

Mu Alpha Theta Sponsor
East Central Jr. College
Decatur, MS 39327

Congratulations!

Your application for a charter in Mu Alpha Theta, international high school and junior college mathematics club, has been approved by the national office and Governing Council, and your charter and membership certificates are enclosed.

I wish that each of these letters could be hand written, or that some of us from the Governing Council might come in person to present this charter to you. We feel that the recongnition that the charter and membership certificates will give is a real boost to help create an even greater interest in mathematics, which is, of course, the key to all creative effort in the sciences. Your community has every reason to be proud of your fine mathematics department and of the students who receive national recognition for their superior accomplishments with the granting of their charter memberships. There is every reason to believe that the great scientists of tomorrow will come from the membership of our club.

Please congratulate each charter member for all of us on the Governing Council. Thousands and thousands of high school students will follow, but they are charter members, and this, we think, is of particular importance.

Kindest personal regards and best wishes for success.

Sincerely,

Harold V. Huneke

Harold V. Huneke
Secretary-Treasurer

Mu Alpha Theta
601 Elm Avenue, Room 423
Norman, Oklahoma 73019

Special To

Decatur, MS, November 25, 1981 --- East Central Jr. College has been honored this week by election to Mu Alpha Theta, an international ~~high school and~~ junior college mathematics club.

The announcement was made by Dr. Harold V. Huneke, national secretary-treasurer, To Mu Alpha Theta ~~who is a~~ professor of mathematics at the University of Oklahoma where the national office is located.

Only those schools with excellent mathematics programs can earn membership in the club since all courses in mathematics and the qualifications of the mathematics faculty and students are examined in detail by the club's Governors and National Officers.

To be eligible for membership, minimum requirements are that a student must have completed with distinction at least four semesters of college preparatory mathematics and be enrolled in the fifth semester. He also must have an overall grade average of at least a "B" in all of his high school work.

"Membership in Mu Alpha Theta is the highest honor possible for a high school or junior college student of mathematics," Dr. Huneke said. "Club activities consist of work in areas of mathematics not usually covered in the classroom."

Mu Alpha Theta was founded in 1957 at the University of Oklahoma and has grown to more than 2,000 clubs in 46 all 50 states and Canada, Japan, Puerto Rico, the Canal Zone, Iceland, Turkey, Brazil, and Okinawa.

The club is co-sponsored by the Mathematical Association of America and the National Council of Teachers of Mathematics and has attracted the attention of top mathematics scholars in this country and abroad. Dr. Shelby Harris is the advisor to the club.

Mu Alpha Theta
601 Elm Avenue, Room 423
Norman, Oklahoma 73019

Special To

Decatur, MS, March 25, 1982 --- Ten students at East Central Jr. College have been honored this week by election to Mu Alpha Theta, international high school and junior college mathematics club.

The announcement was made by Dr. Harold V. Huneke, national secretary-treasurer, who is a professor of mathematics at the University of Oklahoma where the national office is located.

Only those schools with excellent mathematics programs can earn membership in the club since all courses in mathematics and the qualifications of the mathematics faculty and students are examined in detail by the club's Governors and National Officers.

To be eligible for membership, minimum requirements are that a student must have completed with distinction at least four semesters of college preparatory mathematics and be enrolled in the fifth semester. He also must have an overall grade average of at least a "B" in all of his high school work.

"Membership in Mu Alpha Theta is the highest honor possible for a high school or junior college student of mathematics," Dr. Huneke said. "Club activities consist of work in areas of mathematics not usually covered in the classroom."

Mu Alpha Theta was founded in 1957 at the University of Oklahoma and has grown to more than 2,000 clubs in 46 states and Canada, Japan, Puerto Rico, the Canal Zone, Iceland, Turkey, Brazil, and Okinawa.

The club is co-sponsored by the Mathematical Association of America and the National Council of Teachers of Mathematics and has attracted the attention of top mathematics scholars in this country and abroad.

ORDER BLANK

for

Membership Certificates

(Please type. Check spelling with care.)

Name of Chapter ______________________________ Date of Order __________
school

School Address ____
street or po box number

____
city state zip code

Falucty Sponsor ____

Date of initiation __________ New members ______ Amount enclosed ($2.00/stu.)$ ______

First name | Middle initial | Last name | Expected year of graduation

Total # of members ______

We hereby certify that the above named students meet all the qualifications for full membership in Mu Alpha Theta and have been duly elected by this chapter.

______________________ Chapter Secretary

______________________ Faculty Advisor

Mail to: Mu Alpha Theta, 601 Elm ave, Rm. 423, Norman, OK 73019

Mu Alpha Theta
601 Elm Avenue, Room 423
Norman, Oklahoma 73019

Special To

Decatur, MS, April 7, 1983 --- Eleven students at East Central Junior College have been honored this week by election to Mu Alpha Theta, international high school and junior college mathematics club.

The announcement was made by Dr. Harold V. Huneke, national secretary-treasurer, who is a professor of mathematics at the University of Oklahoma where the national office is located.

Only those schools with excellent mathematics programs can earn membership in the club since all courses in mathematics and the qualifications of the mathematics faculty and students are examined in detail by the club's Governors and National Officers.

To be eligible for membership, minimum requirements are that a student must have completed with distinction (at least a B average) at least four semesters of college preparatory mathematics and be enrolled in the fifth semester.

"Membership in Mu Alpha Theta is the highest honor possible for a high school or junior college student of mathematics," Dr. Huneke said. "Club activities consist of work in areas of mathematics not usually covered in the classroom."

Mu Alpha Theta was founded in 1957 at the University of Oklahoma and has grown to more than 1,200 active chapters in most of the states and seven foreign countries.

The club is co-sponsored by the Mathematical Association of America and the National Council of Teachers of Mathematics and has attracted the attention of top mathematics scholars in this country and abroad.

EAST CENTRAL JUNIOR COLLEGE

ESTABLISHED 1928

DECATUR, MISS. 39327

DR. C. V. WRIGHT, PRESIDENT
B. J. TUCKER, ACADEMIC DEAN
F. T. RIVES, REGISTRAR
C. D. BRACKEEN, DEAN OF STUDENTS
PERY B. WINEGARDEN. BUSINESS MANAGER

MATHEMATICS

MAT 1313—COLLEGE ALGEBRA—A review of algebraic operations, systems of linear equations, and a study of logarithms, determinants, progressions, binomial theorem, partial fractions, and theory of equations. Three lectures. Three semester hours credit.

MAT 1323—TRIGONOMETRY—The study of solutions of right and oblique triangles, identities, trigonometric equations, and polar and parametric equations. Three lectures. Three semester hours credit.

MAT 1333—FINITE MATHEMATICS—Introduction to symbolic logic, set theory, probability theory, difference equations, linear programming, and game theory with applications oriented toward business decision-making and the behavioral sciences. Three lectures. Three semester hours credit.

MAT 1423—FUNDAMENTALS OF MATHEMATICS—A review of basic algebra, systems of linear equations and systems of linear inequalities, and introduction to linear programming, and introduction to vector and matric algebra. Three lectures. Three semester hours credit.

MAT 1613—CALCULUS I—Coordinate systems, basic theorems of analytics, functions, limits, the derivative, the integral, differentiation and integration of algebraic functions, and applications. Three lectures. Three semester hours credit. (1823)

MAT 1623—CALCULUS II—Differentiation and integration of transcendental functions, the difinite integral, methods of integration, applications. Three lectures. Three semester hours credit. Prerequisite: MAT 1613.

MAT 1723—THE REAL NUMBER SYSTEM—Structure and properties of number systems of arithmetic. Limited to students preparing to teach. Three lectures. Three semester hours credit.

MAT 1733—INFORMAL GEOMETRY AND ALGEBRA—Basic ideas and structure of algebra; intuitive foundations of geometry. Three lectures. Three semester hours credit.

MAT 2613—CALCULUS III—Solid analytics vector, improper integrals, line integration. Three lectures. Three semester hours credit. Prerequisite: 1623

MAT 2623—CALCULUS IV—Infinite series, partial differentitation, multiple integrals. Three lectures. Three semester hours credit. Prerequisite: MAT 2613

MAT 2913—DIFFERENTIAL EQUATIONS—Solution of first and higher order differential equations; existence theorems; solution by series; and application to problems in geometry; physics and chemistry. Three lectures. Three semester hours credit. Prerequisite MAT 2623.

DIVISION

COURSE, COURSE NAME, PREFIX AND NO.	TEXT
College Algebra MAT 1313	Rees & Sparks; College Algebra: 6th ed., McGraw-Hill Pub. Co.
Trigonometry MAT 1323	Rees & Sparks; Plane Trigonometry: (with tables) 7th ed., Prentice-Hall Pub. Co.
Finite Mathematics MAT 1333	Kemeny, Snell, & Thompson; Introduction to Finite Mathematics: 2nd ed., Prentice-Hall
Fundamentals of Mathematics MAT 1423	Mueller, Elements of Algebra, Prentice-Hall
Calculus I, II, & III, &IV	Riddle; Calculus and Analytic Geometry: 2nd ed., Wadsworth
Mathematics for Teachers I MAT 1723	Ohmer, Ancoin, Cortez; Elementary Contemporary Mathematics: 2nd ed., Blaisdell, A Div. of Gin & Co.
Mathematics for Teachers II MAT 1733	Young & Bush; Geometry for Elementary Teachers: Holden Day Publishing Co.
Differential Equations MAT 2253	Rainville & Bedient; A Short Course in Differentia Equations: 5th ed., MacMillan

NATIONAL HIGH SCHOOL AND JUNIOR COLLEGE

Mathematics Club

Mu Alpha Theta (M A Th)

This is your invitation to establish a chapter of The National High School and Junior College Mathematics Club, M A Th, in your school. The club, which is co-sponsored by the Mathematical Association of America, and the National Council of Teachers of Mathematics is a non secret organization whose purpose is to stimulate interest in mathematics by providing public recognition of superior mathematical scholarship, and by promoting various mathematical activities. Its journal, the Mathematical Log, is constantly being praised as an outstanding medium for up-to-date, understandable articles about Mathematics. There are now over 1200 chapters in 46 states and in Canada, Japan, the Canal Zone, and Switzerland. If your school can qualify you owe it to your students to establish a chapter and to participate in their many stimulating activities.

I. Purpose of the Organization

The National High School and Junior College Mathematics Club, Mu Alpha Theta was formed to engender keener interest in mathematics, to develop sound scholarship in the subject and promote enjoyment of mathematics among high school and junior college students.

National policy is determined by a nine-person governing council consisting of three national officers (president, president-elect or immediate past president, and secretary-treasurer), four governors elected for three-year terms, and representatives appointed by the Mathematical Association of America and the National Council of Teachers of Mathematics. These organizations each have the privilege of nominating two of the seven elected council members. Others are nominated by a committee of the club. Officers and governors serve without remuneration.

The governing council provides the following services for chapters:

1. Individual Membership Certificates, billfold cards and School Charters are issued by the national Secretary-Treasurer.
2. The Mathematical Log, official journal of the organization, is distributed free to the chapters. It is devoted to understandable articles on mathematics, both modern and ancient, which have been especially written for the intended audience. The Log also contains current chapter news, news of regional and national meetings, suggestions of topics suitable for discussion at chapter meetings, discussion of recent books, periodicals, films, tape recordings, mathematical devices and similar items of interest to the membership.
3. One copy of our other publications, now six in number, are sent to each chapter without charge. Additional copies are available at a 40% discount.
4. Up-to-date information about mathematical careers is supplied to each chapter.
5. Local Newspapers are notified that a charter has been granted, and of the names of all students elected as full members.
6. Insignia pins and buttons are available from the National Office.
7. Regional meetings and other activities are constantly being arranged by the chapters.
8. National meetings are held with lectures by outstanding mathematicians, as well as by students.
9. Chapters are encouraged to make suggestions to the national officers concerning additional projects which they would like to see carried out. A sincere effort is made to be as helpful as possible.
10. The Handbook for Sponsors.

II. School Qualifications for Chapters

Any high school, two-year junior college, or other academic institution giving training equivalent to one of these, may petition to have a chapter providng it meets the following minimum requirements:

1. At least six semesters of mathematics including algebra, geometry and more advanced topics (or, in the case of junior colleges, courses including the calculus) must be offered. These requirements can not be fulfilled by courses in general mathematics, business mathematics, shop mathematics, or arithmetic.

2. During the two semesters preceding that in which a petition is submitted, the school must have employed at least one teacher whose primary teaching field is mathematics and who has completed an undergraduate mathematics major or its equivalent at an accredited college or university.

3. The Principal, or other chief administrative officer of the institution, must approve the petition.

4. An initial charter fee of $2.00, along with the regular initiation fee of $1.00 for each member, must accompany the petition for a charter. (These fees will be returned if the governing council votes that the institution is ineligible for membership.)

5. A favorable vote of two-thirds of the governing council shall be required to elect a chapter to membership. Each petitioning institution will be notified as soon as possible whether or not a charter has been granted. This takes about 3 weeks.

6. The petition should be sumbitted (along with 10 copies) on the enclosed form provided by the national office. The copies may be on plain paper.

III. Qualifications for Individual Membership

The following minimum requirements for full and associate membership shall be common to all chapters. Each chapter shall have a faculty-student committee which will recommend possible members for the chapter's consideration. No student shall be recommended for consideration who does not meet the minimum qualifications. Additional requirements may be imposed by individual chapters.

(a) Full Membership: High school students who have completed the equivalent of four semesters of college preparatory mathematics and in addition have completed or are enrolled in a still more advanced course, are eligible for full membership providing their mathematical work was done with distinction (on the ABCDF grading scale this shall mean at least a B average).

Junior College students are not eligible on the basis of their high school records alone: they must have in addition a B average in the equivalent of at least one semester of college mathematics which has as a prerequisite the equivalent of five semesters of college preparatory mathematics. Junior College members, however, who already were elected in high school continue to be members and are eligible to belong to a junior college chapter.

(b) Associate Membership: Persons who have completed two semesters of algebra or their equivalent with distinction and who have enrolled in a third semester of mathematics are eligible for associate membership. Associate members do not pay the $1.00 initiation fee and are not registered with national headquarters. They are not entitled to vote on national policy. They are entitled to attend and be heard at meetings, and presumably are likely candidates for full membership.

IV. National Finances

There shall be no national annual dues. An initiation fee of $1.00 per full member shall be paid to the national secretary-treasurer for each person initiated into full membership, where-upon the secretary-treasurer shall issue a membership certificate to that member and place his name on the official roll. A $2.00 charter fee shall be charged each new chapter at the time the official charter is first issued to the school.

Finances are supervised by the Controller's Office of The University of Oklahoma. Regular state supervised auditing service is provided, in addition to approved monthly machine accounting. Council approval is requied for expenditures other than printing and metered postage, and the secretary-treasurer must make complete accounting to the Council.

V. Local Organizations, Officers, and Finances

Each chapter is free to set up its own organization, officers, and finances with the following restrictions:

1. Each chapter must have a semi-permanent faculty-sponsor and the national office (secretary-treasurer) must be kept informed of the current faculty sponsor's name and address.

2. The minimum membership requirements set forth by the national office must be met by all initiates.

3. A complete and accurate list of all full member initiates, accompanied by their $1.00 initiation fees, must be received by the office of the secretary-treasurer before the initiation takes place.

4. Each chapter must hold regular meetings at periodic intervals and not merely consider itself an honor society for high grades. At the minimum, there should be one meeting every month.

5. Local chapters are encouraged to participate actively in the life of the school, providing stimulation of an interest in, and appreciation of, mathematics for all students.

6. Either the name National High School and Junior College Mathematics Club or Mu Alpha Theta may be used as desired.

VI. Governing Council

PRESIDENT: Ms. Katherine P. Layton, Beverly Hills High School, Beverly Hills, CA 90212
PAST PRESIDENT: Dr. Robert Kalin, Florida State University, Tallahassee, FL 32306
SECRETARY-TREASURER: Dr. Harold V. Huneke, University of Oklahoma, Norman, OK 73019
OFFICIAL REPRESENTATIVE of the MATHEMATICAL ASSOCIATION of AMERICA:
Dr. Robert Wilson, Ohio Wesleyan University, Delaware, OH 43015
OFFICIAL REPRESENTATIVE of the NATIONAL COUNCIL of TEACHERS of MATHEMATICS:
Mr. Alvin Gloor, Westside High School, Omaha, NB 68144

GOVERNORS:

Region I Mr. James Woolum, Clayton Valley High School, Concord, CA 94521
Region II Ms. Kathryn Hinsenbrock, Community High School, Charles City, IA 50616
Region III Ms. Adele Hanson, Milwaukee Technical High School, Milwaukee, WI 53204
Region IV Mr. Thomas Thrasher, Austin High School, Decatur, AL 35601

THE MATHEMATICAL LOG

Volume XIX, No. 3 April, 1975

AN EXCURSION INTO THE NON-EUCLIDEAN WORLD

Say there, you folks, did you ever meet anyone out of this world? Let me tell you! You could be out of this world, yourself, if you took off with me on a trip to the world of Henri Poincare. Further than the moon and closer than the stars, this non-Euclidean space has attractions that appeal to the majority of us who have been ravaged by a sub-zero winter, a spring freeze, a minus sunshine factor, and an energy crisis. Try to picture a land where there is no recession, no inflation, no depression. Fantastic, isn't it?

Before I tell you about this non-Euclidean World, I want to call your attention to the vagueness of such fundamental notions as straight line, plane and distance. Take, for example, the "distance between two points A and B." What does that mean? We think of measuring the distance, say with a foot rule, but that doesn't help much. A ruler is divided into equal intervals and hence presupposes the notion of distance. But what about equal distances? The distance, whatever it is, between A and B is equal to the distance between points C and D if the pair of points A and B can be shifted without changing their mutual distance, so as to coincide with the points C and D. But again the difficulty--the condition "without changing their mutual distance" implies that we know what is meant by equal distance.

We[illegible] join the points A and B by a straight line. Su[illegible] we can say that the distance AB is equal to the distance CD, if the segment of the line AB can, by a rigid motion of this segment, be made to coincide with the segment joining C and D.

But does the notion of distance between two points necessarily involve the notion of straight line? Doesn't an intoxicated person cover distance between the bar and the swinging doors? And he certainly doesn't walk a straight line. And what is this rigid motion you speak about? Just think for a moment--isn't the notion of distance rather complicated? The imaginary world on which we are about to land will make us realize more vividly how serious these difficulties of distance really are.

This imaginary world is enclosed entirely within a large three-dimensional space whose boundary is a large spherical surface with radius R. The temperature here is rather fanciful. It changes from point to point. At the center it is a maximum and decreases gradually until it is absolute zero at the boundary. Suppose we are at a distance r from the center of the sphere, then t , the temperature is $(R^2 - r^2)$. When r - 0, at the very center, $t = R^2$, a maximum. When $r = R$, then $t = 0$, the temperature at the boundary. Another point: the temperature is constant on the surface of a sphere within our world provided this sphere is concentric with the world.

The inhabitants and material objects change with the temperature, growing larger or smaller in size in direct proportion to the change in temperature. Therefore, if a man walks toward the center he gets larger, towards the boundary he gets smaller. These changes, by-the-way, are instantaneous.

Wi[illegible]is description in mind, we ought to begin to see some of the properties of the geometry which a man living in that world would develop. To him it would seem to be of infinite extent, although to us, it seems to be finite. As I mentioned before, if he started to walk toward the boundary, as the temperature fell, his body would grow smaller and his steps gradually shorter, contracting indefinitely as he approached the surface of the bounding sphere. To reach the boundary of his world, he would have to take an infinite number of steps, since $1/2 \log \frac{1 + x}{1 - x} \longrightarrow \infty$

The question naturally arises whether he would notice how bodies changed in size when their distance from the center of the world changed. The way in which we usually compare the sizes of objects is to place them side by side, or measure them, say with a ruler or tape. If he moved from one part of his world to another and took a beef burger with him, it would change at the same time and in the same proportion as he changed. It is obvious that the man would never realize that his beef burger is getting smaller. He is getting smaller himself.

In some respects his geometry would resemble our own, but in others it would differ. Suppose, for instance, the man wished to go from his house (H) to the dog house (DH) by the smallest number of steps. It is reasonable to suppose that the smallest number of steps would not be taken along a straight line HDH joining the house and the dog house, but rather along a path (say HnDH) which swerves toward the center, since his steps would be longer there. In fact, it can be rigorously proven, by the calculus of variations, that the path which gives the smallest number of steps is the arc of a circle which cuts the bounding sphere orthogonally. This fact has been proved previously in the literature. Such a circle we shall call a "shortest line" or "geodesic". The man would certainly think of it as our own "straight line." In another respect, this arc or shortest line has the same property as our straight line, namely, that between two points there can be only one straight line. Let us prove this.

Let us take a cross section of this sphere through center which will be the circle ABC. P is any point in the plane of the section and within the circle. Through P draw the diameter DD', also a line perpendicular to it through P, which will intersect the bounding circle in a pair of points, say T and T'. The tangents to the circle at T and T' will intersect upon the diameter, at a point which we call P'. This can easily be shown in plane geometry. (If two triangles have two sides of the one equal, respectively, to two sides of the other, and the angles opposite two equal sides equal, the angle opposite the other two equal sides are equal or supplementary, and if equal the triangles are equal.) Then any circle through P and P' will cut the given sphere orthogonally; and, conversely, any circle cutting the given circle orthogonally and passing through P will also pass through P'. It follows readily that through any two points within the sphere there is one and only one shortest line.

Proof:

> OTP' is by construction a right triangle, so that for any point P and the corresponding point P' we always have $OP \cdot OP' = OD^2$. Let QPP' be any circle through P and P', C its center, Q' the other extremity of the diameter through O. Then let QPP' cut circle ABC at M. Hence, $OM^2 = OQ \cdot OQ' = (OC - QC)(OC + QC) = OC^2 - QC^2 = OC^2 - MC^2$. Whence $OM^2 + MC^2 = OC^2$, and OMC is a right angle.

There is one respect, however, in which the geometry of the shortest line would differ from the geometry of our straight line.

From his parallel postulate, Euclid derived the theorem, that through a given point P not on a given line L, there is only one line parallel to L. One way of describing this notion is to draw any line through P cutting L in a point which we will call Q. If the point Q moves in either direction along L, the line PQ will at the same time revolve about the point P. The farther Q proceeds along L, in one of the two possible directions, the nearer will PQ approach a limiting position. Since there are two directions in which Q can move without limit along L, there are two ways of

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THE MATHEMATICAL LOG

April, 1975 Volume XIX, No. 3

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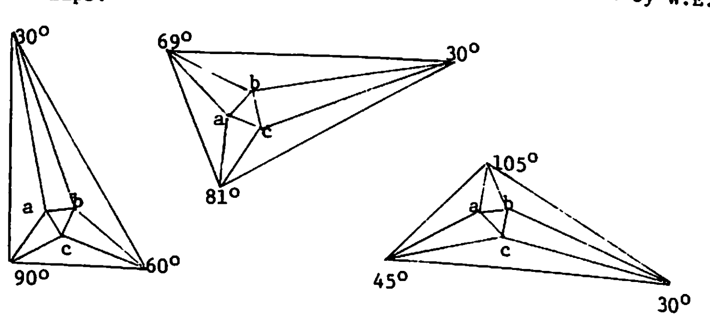
Calling Yahtzee fans...Ever wondered what your chances of getting a pair if you got three of a kind on the first roll? 1/4, 1/40 or 1/400?

You think you understand long numbers? Well then, how long would the answer to this evaluation be?

9^{9^9} We may assume you can write five digets to the inch. 30 in., 30 ft., 30 mi., none of these.

Here's one we're sure you didn't know! If you trisect the angles of a triangle (We know you can't do it by construction..use a protractor) and name the intersection of the rays that are adjacent to the side the triangle A,B,C, the triangle formed by connecting A, B, and C will always be equilateral. Honest. Try it. The property was first discovered by F. Morley around 1900, but it was not proven until 1914 by W.E. Phillips.



SIXTH NATIONAL CONVENTION-AUGUST, 1975
SEATTLE UNIVERSITY
SEATTLE, WASHINGTON

The program will feature Charles Allen, Director of NCTM, as Banquet Speaker. Other speakers include Roy Dubisch, Calvin Long, Dr. H.M. Cox, Elden Egbers and Jack Robertson. Some topics are: "How A Mathematician Thinks", "Serving PI", "How To Roll A Polygonal Wheel", "The M.A.A. Contest and the Olympiads", "Sperner's Lemma", "Coding and Matrices" and many, many more.

Special tours will include "Underground Seattle", The Pacific Science Center where we will hold a session, Pioneer Square and the Waterfront.

Plan NOW to come and bring a group. For additional details write: Robert Marion, Route 1, Box 1080, North Bend, Washington 98045.

SOME STUDENT TALKS FROM THE FIFTH NATIONAL MU ALPHA THETA MEETING

A FACTORIAL CURIOSITY

Dan Jasicki

Would you believe that there are more possible seating arrangements in a classroom with thirty chairs than there are drops of water in all the oceans of the earth?

The seating arrangement is found by

$$30! = 30\cdot 29! = 30\cdot 29\cdot 28! \cong 2.65 \times 10^{32}$$

Let's agree the standard eyedropper is $1/10\text{cm}^3$.

$$1 \text{ drop} = 1/10\text{cm}^3$$

$$10 \text{ drops} = 1 \text{ cm}^3$$

$$10(100)^3 = 10^7 \text{ drops} = 1(\text{meter})^3$$

$$10^7(1000)^3 = 10^{16} \text{ drops} = 1(\text{kilometer})^3$$

Radius of earth is about 3959 miles or 6.37×10^3 kilometers.

If the earth were spherical its volume would be

$$(4/3)\ r^3 = (1.333)(3.142)(6.37 \times 10^3)^3 = 1.08 \times 10^{12} \text{ cubic}$$

kilometers. Then the total number of drops would be...

$$(1.08 \times 10^{12}) \times 10^{16} = 1.08 \times 10^{28}$$ if the earth were all water!

Hey! Wait a minute! Our calculations show there are about 15,000 more seating arrangements than drops of water even if the whole world was water.

THE PERFECT SHUFFLE

Terry Ligocki

My topic deals with a "perfect" shuffle. By this I mean I take a deck of fifty-two cards and cut it into two piles of twenty-six cards each, then the bottom card of the bottom pack goes down first, then the bottom card of the top pack goes on top of that and so on (this will be demonstrated). One practical application of this is a line of cheerleaders [Dia. 1-(1)] who divide in half [Dia. 1-(2)], then both halves move toward one another [Dia. 1-(3)] and merge [Dia. 1-(4)]. In the third part [Dia. 1-(3)] the cheerleaders are turning over their signs and the letters on the right are the new letters which were originally face down.

If a deck has an even number (2n) of cards the new position of a card in the top half (a ≤ n) can be predicted by doubling its position (a) and subtracting one (2a-1). In the bottom half (a > n) a card's new position can be predicted by doubling its position (a) and subtracting the number of cards in the entire deck (2a - 2n or 2(a - n) using the distributive property). As you will note, using the formula I have given you the first and last cards of any "even" deck don't move, so there are 2n - 2 "working" cards which is 2(n - 1) which is an even number.

If the number of cards in a deck is odd (2n - 1) then it can be divided in two ways--n cards on the top and n-1 on the bottom [Dia. 2-(A)] , or n-1 on the top and n on the bottom [Dia. 2-(B)]--which means it can be shuffled two ways (Dia. 2).

To predict the position of a card in an "odd" deck, as in Dia. 2-(A) or-(B), the formula for the cards in the top "half" of the deck is the same as the one used for an "even" deck. The formula for the cards in the bottom "half" is 2(a-the number of cards in the top "half" of the deck (n or n-1)), except for the very bottom card in Dia. 2-(B) (labelled "e") which uses the special formula [2•(a - n - 1)] - 1 = [2•(a - n + 1)] - 1 = (2a - 2n + 1 2(a - n) + 1 because it is an "extra" card as you can see from the diagram.

In Dia. 2-(A) only the top card doesn't move so there are (2n - 1) - 1 "working" cards which is 2(n - 1) which is an even number. In Dia. 2-(B) the top card and the two very bottom cards don't move so there are (2n - 1) - 3 "working" cards which is 2(n - 2) which again is an even number of working cards.

continued on page 3

I leave it to you to decide why the number of "working" cards is always even, to explore other shuffles, and to predict the length and the number of loops cards go through--for example with a deck of ten cards the loops are 2nd, 3rd, 5th, 9th, 8th, 6th,and back to 2nd,and 4th to 7th and back to 4th; then, with a deck of twelve cards there is only one loop ten cards long which is 2nd, 3rd, 5th, 9th, 6th, 11th, 10th, 8th, 4th, 7th, and back to 2nd.

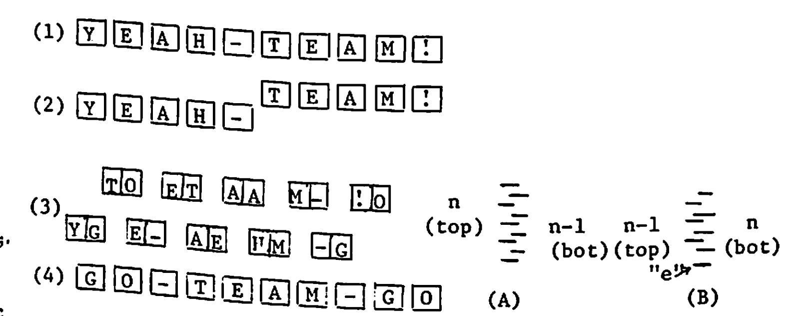


DIAGRAM 1 DIAGRAM 2

"1089"

Frank Gorenc

Take any three digit number whose digits don't repeat. Turn the number around and subtract the smaller from the larger. Take the number you get and turn it around and add them. You will get 1089.

Example:

$$\begin{array}{r} 321 \\ \underline{123} \\ 198 \\ \underline{891} \\ \text{"1089"} \end{array}$$

Proof:

(Subtract) $\begin{array}{r} 100a + 10b + c \\ \underline{100c + 10b + a} \\ 99a - 99c = 99\cdot(a-c) \end{array}$

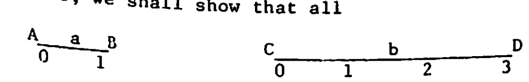
Let e = (a-c) then (100-1)e = 100e + 0 - e

$$\begin{array}{l} 100e + 0 - e + 100 - 100 \\ 100e - 100 + 90 + 10 - e \\ 100\cdot(e-1) + 10\cdot(9) + 1\cdot(10-e) \\ \underline{100\cdot(10-e) + 10\cdot(9) + 1\cdot(e-1)} \\ \qquad - 10 \\ 100e - 100 + 90 + 10 - e \\ \underline{1000-100e \qquad + 90 \qquad + e - 1} \\ 1000 \qquad + 80 \qquad + 9 \\ \text{"1089"} \end{array}$$

Frank Gorenc

THE EQUIVALENCE OF THE UNIT LINE SEGMENT AND THE UNIT SQUARE

When dealing with infinite sets our usual intuitive feelings no longer hold. For example, we shall show that the line segment b which is 3 times as long as line segment a is "equivalent" to a. That is, we shall show that all

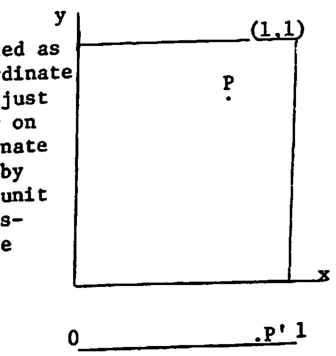


the points of a may be matched with all the points of b , in a one-to-one manner. Whenever there exists a one-to-one match or correspondence between two sets we say that the sets are equivalent. We are going to show that a and b are equivalent when regarded as sets of points. If P is any point of a it has associated with it a number n between 0 and 1: 0 ≤ n ≤ 1. We say that the coordinate of P is n. The point P' having the coordinate 3n in b will be the point corresponding to P. It works the other way too. For any point in b we find its corresponding point in a by dividing its coordinate by 3 and locating the point having this number for its coordinate in a. The correspondence between the points of a and b may be shown by

n —— 3n

This correspondence proves that a and b are equivalent when regarded as sets of points. Of course, b could have been 10 units long, or 100, or a billion and a very similar proof can be made. In fact, a and b could have any lengths at all and in a similar manner we could show that a and b are equivalent when regarded as sets of points.

Now to show the equivalence of a unit line segment and a unit square. We have three parts to our presentation. In the first part the proof will have two flaws. You should stop and try to find them. In the second part we correct one of these flaws. In the third part we point out a fundamental flaw in our argument, one that you may have overlooked, and then show how it may be corrected. This will correct all the errors and the proof will be complete.



Part 1 We take our unit square located as shown in the first quadrant of a coordinate system. We show a unit line segment just below it. Let P be any point in or on the unit square. Denote its x-coordinate by $.a_1a_2a_3\ldots$ and its y-coordinate by $.b_1b_2b_3\ldots$ To find the point in the unit line segment, which we call P' corresponding to P , we find the coordinate of P' by intermeshing the x and y digits for P , getting

$$.a_1b_1a_2b_2a_3b_3\ldots$$

Now suppose that we want to find the point corresponding to any point P' in the unit line segment. Suppose that the coordinate of P' is

$$.c_1c_2c_3c_4c_5c_6\ldots$$

We simply reverse our meshing procedure to obtain the x and y coordinates:

$$x = .c_1c_3c_5\ldots \qquad y = .c_2c_4c_6\ldots$$

The mate of P' is P having (x,y) for its coordinates. Try to find the two flaws in the above argument.

Part 2 One of the difficulties we have to correct is that, except for 0, all terminating decimals have equivalent non-terminating decimals. For example:

$$.5 = .4999\ldots, \qquad .72 = .7199\ldots$$

When a coordinate has a terminating decimal we must decide whether to use it or its equivalent non-terminating decimal. To make things more uniform, we agree to always use the non-terminating form for this number, except for 0. For example,

In place of	We use
.5	.4999...
.72	.71999...
1.0	.999...
0.0	.000...

The only terminating decimal we are permitted to use is the one for 0, but even for 0 we shall use .000... With this agreement we have a unique way of obtaining the correspondence, supposedly. There is another flaw and a fundamental one, too. Have you found it?

Part 3 Strangely enough, with our unique way of naming numbers we actually have shown how to find the mate of every point in and on our unit square. It does not work from the line segment to the square. There are infinitely many points of the line segment that do not have mates in the square. Here is one example of a point that has no mate in the square:

.5101010101...

continued on page 4

If we split it we obtain for the x-coordinate .5000... which is unpermissible. We agreed to use in place of this numeral .499.... There are infinitely many such cases, but one is enough to show that we do NOT have, according to this approach, a one-to-one correspondence between the points of the unit square and the unit line segment. In fact, we do have a one-to-one correspondence between all the points of the unit square and a subset of the points of the unit line segment. To complete the proof we may invoke an important theorem which we shall state without proof. It is the Schroeder-Bernstein Theorem:

> If set A is equivalent to a subset of set B, and B is equivalent to a subset of A then sets A and B are equivalent.

Clearly, the unit line segment is equivalent to a subset of the square.

Our proof is complete. Do you see how the proof may be modified to show that a unit line segment is equivalent to a unit cube? Can you extend this further?

If you would like more information on infinite sets you may want to read one or more of the following:

"Theory of Sets" by Kamke, and translated by Frederick Bagemihl, published by Dover Publications, Inc. New York

"Introduction to the Theory of Sets" by Joseph Breuer, and translated by Howard F. Fehr, published by Prentice-Hall, New Jersey

"Naive Set Theory" by Paul R. Halmos, published by D. Van Nostrand

You need some help?

Yahtzee fans.....5/36 chances on the second roll, OK? and (31/36) (5/36) on the third roll for a probability of 335/1296 or approximately 1/4. Ohhh, you knew that!

$\log 9^{9^9} = 9^9 \log 9 = (387,420, 489) \cdot (.95424) = 369692127.4$
antilog $= 2.512 \times 10^{369692127}$

therefore 9^{9^9} has 369692127 digits in the answer and at the rate of 5 digits in an inch the answer would be 1167 miles long!! We knew you didn't know that.

The suggested proof for Morley's Theorem can be found on page 544 in Geometry-Fundamental Structure, Scott Foresman, 1965. (The complete solution is in the Teachers Commentary, page 115.)

AN EXCURSION INTO THE NON-EUCLIDEAN WORLD

obtaining limiting positions of the line PQ. Euclid's fifth postulate implies that these two limiting positions are the same, but, of course, our geometry need not be Euclidean.

Let us see what proposition the people in the "spherical world" would have corresponding to the proposition just quoted, if we think of the shortest lines playing for them the role of straight lines. We shall see that they would have no use for Euclid's postulate, and that as a statement of a general truth it would even appear ridiculous. Corresponding to the line L, they would have a circle L, meeting the boundary of the world at right angles. Through a point PP, not on L, they would construct another shortest line intersecting L at a point Q, then let Q travel off in either direction along L. For each position of Q there would be a definite circle PQ cutting the boundary orthogonally. Let us see what the limiting positions of PQ would be.

To the inhabitants of the sphere, Q would appear to be approaching the point R on the boundary, where L intersects it. But since their world would seem infinite to them, and no material object could ever reach its boundary, it would seem to them that the point Q could never reach R. They would consider PR a limiting position of PQ. By allowing Q to travel in the opposite direction, they would obtain a second limiting position PS. These two circles do not, in general, coincide. The angle θ, which they make with each other, is in general so appreciable that the inhabitants would easily observe it. Defining two shortest lines to be parallel, if they are in the same plane and do not meet, however far they are produced, we see that the inhabitants of this world would recognize the existence of an infinite number of shortest lines through P and parallel to L, namely, all such lines lying within the angle θ. The two limiting parallels PR and PS we call the principal parallels. On account of these contradictions to Euclidean geometry, we call this new geometry non-Euclidean.

Let us consider the angle θ. We notice that this angle decrease as the point P and the line L approach the center of the sphere, and may be made as small as we please by taking P and L sufficiently close to the center. We must remember that "sufficiently close" must be understood relatively to the radius of the bounding sphere. It is perhaps better to put it in this way: If P and L are at a given distance from the center, the angle θ decreases indefinitely as the radius of the bounding sphere is increased and may be made as small as we please by making this radius sufficiently large.

Now suppose our earth were at the center of such a sphere (concentric), and suppose the radius of the latter is assumed to be so large that in all the space about the earth which is accessible to our observation, the angle that above mentioned is smaller than any instrument of ours can detect. The observations on the space in which we would thus find ourselves would differ in no particular from those to which we are accustomed; everything would look and feel just the same. But--and this is the point that is to be emphasized--the abstract non-Euclidean geometry may be applied to these observations quite as legitimately as the Euclidean. If we keep in mind the distinction between an abstract science its concrete application, we see there is absolutely no way of telling whether we live in a Euclidean or in a non-Euclidean world. We do know that the angle θ, if it exists, is so small that we cannot detect it. So, for all practical purposes, Euclidean geometry is the more convenient to use, and there can be no doubt that it is the form of geometry which will always be used in practical applications. What we wish to emphasize is that the idea of there being more than one parallel to a line through a given point is in no way inconceivable, and in no way contradicts anything that we observe in our world. Henri Poincare, who thought of such a world, and Lobachewsky, who challenged the fifth postulate of Euclid, have given wonderful geometry to men. To study it is a rare experience.

You shall hear the question asked, "Which is the true geometry?" We can see now that the question cannot be answered for it is without meaning. It is very much as if we were to ask "Which is true, the measuring system using rulers marked off in feet and inches or marked off in centimeters?" One might ask whether it is more convenient to measure things in feet and inches than it is to measure them in centimeters. Non-Euclidean geometry is much more complex than Euclidean geometry, and as Poincare remarks, "Euclidean geometry is, and will remain, the more convenient." However, both are valid.

Whether our readers are in high school, college or in continuing education, the notions stressed in this paper are provocative. Young high school students should be introduced to the geometry of our fathers. However, there are many other ways of considering this branch of mathematics. For collegians a lecture sometime during the year will set at ease those students who have heard about non-Euclidean geometry. Keep in mind that all geometry is dependent on arbitrary (though reasonable) assumptions and hypothesis and that the changing of a postulate may produce different theorems.

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THE MATHEMATICAL LOG

Volume XXII, No. 1 October, 1977

1 2 3 4 5 6 7 8 9

WOMEN IN MATHEMATICS: BIOGRAPHICAL SKETCHES

The study and practice of mathematics has been regarded popularly as male territory. In actuality, the "women's liberation movement" in mathematics is not something recent. It has been evolving for many centuries. Through the years, women have had to overcome much social pressure and actual discrimination in order to achieve some measure of recognition for their mathematical efforts. Who were these women who dared to invade mathematics? What problems did they encounter? What did they achieve? This material has been written in an attempt to answer such questions and to spark further interest in feminine mathematical contributions.

SOPHIE GERMAIN

Known as one of the founders of mathematical physics as well as a philosopher, Marie-Sophie Germain was born in Paris on April 1, 1776, the daughter of Ambroise-Francois Germain and Marie-Madeleine Gruguelu. The family was prosperous, her father being a goldsmith by trade and later, the director of a bank. Growing up during the political upheavals of late eighteenth-century France, she had to circumvent many extant prejudices against women scholars in general and women mathematicians in particular.

Her initial mathematical training was self-acquired, much against the wishes of her parents. Despite the fact that the newly-founded École Polytechnique did not formally admit women students, Sophie managed to obtain lecture notes of courses taught. She became particularly interested in the mathematical analysis of Lagrange. Using the name of M. le Blanc, she submitted a paper to Lagrange which impressed him. He ascertained her identity and became her mathematical mentor. She also used the same pseudonym in later correspondence with Gauss on number theory problems. Most of her advanced training was acquired by means of correspondence with scholars of the day rather than by formal class attendance.

Sophie's early research was in number theory. She also became interested in a problem involving vibrations of elastic surfaces. Her first paper on this topic, submitted anonymously in 1811 to the French Academy of Sciences, was flawed. She continued to work on this problem, however, winning the grand prize offered by the Academy in 1816 for her "Memoir sur vibrations des lames élastiques". She was honored publicly for her achievement and was invited to attend sessions of the Academy. Sophie published several additional papers dealing with the theory of elasticity, the last of which appeared posthumously in 1831.

Sophie's interests were not restricted to mathematics, however. She studied philosophy, outlining her views in "Considérations générales sur l'état des sciences et des lettres aux différentes époques de leur culture". The physical sciences and history also attracted her attention.

In recognition of her mathematical talent, Gauss recommended that the University of Göttingen grant her an honorary doctorate degree. Before the degree could be conferred, however, she died of cancer in Paris at the age of 55 on June 27, 1831.

SONYA KORVIN-KRUKOVSKY KOVALEVSKY

Another woman who had to scheme and plot to obtain a thorough mathematical education and an appropriate position was Sonya Kovalevsky. Born in Moscow on January 15, 1850, Sonya Korvin-Krukovsky was the daughter of Vasily Korvin-Krukovsky and Yelizaveta Shubert, the middle child of three. Her parents were members of the Russian nobility, the father being a general in the army. Despite the fact that both her grandfather and great-grandfather were mathematicians, she had to overcome strong parental opposition to a mathematical career.

Her early training in mathematics was under the tutelage of Strannolyubsky at the naval academy in St. Petersburg. Since Russian universities were closed to women students, Sonya was forced to go to a foreign university for further training. To gain freedom to travel, she contracted a marriage of convenience with Vladimir Kovalevsky. The couple went to live in Heidelberg where Sonya attended classes in both mathematics and physics but was not permitted to matriculate. Her teachers included the mathematicians Königsberger and Du Bois-Reymond as well as the physicists Kirchoff and Helmholtz. Her desire to study at the University of Berlin under Weierstrass was foiled by the prohibition against women students. To circumvent this, Sonya went directly to Weierstrass and asked him to give her private instruction. When he discovered that even he could not change university policy, he became her private tutor for four years discussing with her not only his lecture notes but also many of his as yet unpublished mathematical ideas.

During this period of study with Weierstrass, Sonya wrote three research papers including her dissertation, entitled "Zur Theorie der partiellen Differentialgleichungen ". The other two papers were published -- "Über die Reduction einer bestimmten Klasse Abelscher Integrale dritten Ranges auf elliptische Integrale" and "zusätze und Bemerkungen zu Laplaces Untersuchungen über die Gestalt der Saturnsringe". In 1874, Sonya was granted her doctorate degree without examination, summa cum laude, by the University of Gottengen. It was conferred in absentia because she was a woman.

Weierstrass attempted to find a position for her but again, the fact that she was a woman, stood in her way. She returned to Moscow and busied herself for almost three years with various literary endeavours--writing newspaper articles, poetry, and drama criticism. With the passing of time, Sonya became more and more unhappy and decided to return to Berlin on her own. Nevertheless, her husband's subsequent suicide, in the spring of 1883, affected her deeply.

While in Berlin, she worked with Weierstrass on the refraction of light in crystalline medium. The results were presented at a scientific congress held in Odessa in September 1883. Through the intercession of Gosta Mittag-Leffler, another student of Weierstrass, Sonya obtained an appointment at the University of Stockholm to lecture on the theory of partial differential equations. Initially, she was an unpaid lecturer (1883-4). Her title was then changed to Professor of Higher Mathematics. In 1889, she was granted a life professorship. She proved to be a popular, effective, and inspirational teacher. The courses she gave covered a broad spectrum of then known mathematics. She also served as editor of the journal ACTA MATHEMATICA which was founded by Mittag-Leffler in 1882.

Sonya continued doing research and in 1888 won the Prix Bordin from the French Academy of Science for her memoir entitled "Sur le probléme de la rotation d'un corps solide autour d'un point fixe". In 1889, she received a prize from the Stockholm Academy for additional research on the same problem. That same year, she was elected to membership in the Russian Academy of Sciences, the first woman so honored.

Sonya's interest in literature also continued. Her published novels include The Sisters Rajevsky, an account of her childhood recollections, and Vera Vorontzoff, (or The Nihilist), a description of life in Russia.

Unfortunately, the last years of her life were not particularly happy from a personal standpoint. Her older sister had died suddenly in 1887. She became involved in

(cont. Pg. 4, col. 1)

The official publication of the National High School and Junior College Mathematics Club, Mu Alpha Theta, which is sponsored by the Mathematical Association of America and the National Council of Teachers of Mathematics. Addresss correspondence to: Mu Alpha Theta, 601 Elm Avenue, Room 423, The University of Oklahoma, Norman, Oklahoma 73019.

MESSAGE FROM THE PRESIDENT

If the activities of our student members at the recent National Convention in Dubuque, Iowa, are at all indicative, then Mu Alpha Theta has had another successful year.

There are several actions I'd like to suggest we take so that this coming year may be equally successful. My first suggestion arises from a recent action of your Governing Council - to give our clubs and student members a chance to influence national Mu Alpha Theta policy, the Council has established a Delegate Assembly of students, to be convened at each National Convention. Each club should now select one delegate, sending his or her name to the National Office. Having thus been formed, the Delegate Assembly shall have its first two-hour meeting at our eighth annual convention at Stevens Point, Wisconsin, next August. At that time, in addition to whatever other recommendations it may suggest and business it may transact, the Assembly will give delegates from each of our four Regions an opportunity to select a representative to a newly formed year-long Student Advisory Board to the Governing Council.

Elsewhere in this issue of the Log is an announcement from the Nominating Committee for the selection of a new President-Elect and a Governor for each of Regions III and IV. I urge student members and sponsors to take part in the nominating process by recommending names of sponsors or other mathematics teachers who have contributed wisely and consistently to the welfare of our clubs and to mathematics instruction in general.

Mu Alpha Theta has always been proud of its sponsorship of the High School Mathematics Contest. This contest has in turn led to our support of the U.S.A. Olympiad, and to American participation in the International Olympiad. So I am positive that all Mu Alpha Theta members take particular pride in the USA team's taking first place in this past year's International Olympiad, and thank not only the student members of this team, but all those who took an active leadership role in their Olympiad participation.

Many sponsors and students have in recent years expressed some concern over the content of the High School Contest. Should you have any feelings about this, make them known to the Governor from your region, the national office, or our Mu Alpha Theta representatives to the High School Mathematics Contest Committee: Dick Pieters of 13659 Preston Rd., #210, Dallas, TX 75240 or Martha Zelinka of 32 Pigeon Hill Rd., Weston, MA 02193. The more specific your suggestion, the easier it will be to understand and implement it. . . . . .Robert Kalin

ANNOUNCEMENTS

The annual mathematics tournament sponsored by the Mu Alpha Theta Chapter of Pecos High School in Pecos, Texas, will be held on January 14, 1978. Kim Hawkins is President of the Pecos Chapter and Caroline Rankin is the Sponsor.

The Eighth Annual Mu Alpha Theta National Convention is scheduled for Stevens Point, Wisconsin, next summer.

The Third Annual Florida Mu Alpha Theta Convention is scheduled for February 17-18, in Miami. The meeting is being jointly planned by three schools, and the Sponsors are Mary Elizabeth Sullivan, Jo Anne Taber, and Hector Hirigoyen.

The Third Annual Tennessee Mu Alpha Theta Convention is scheduled for March 10-11, in Knoxville. Elizabeth Coffin is the State President.

The Annual High School Mathematics Examination has been set for March 14. Registration is handled regionally and closes January 15. A list of regional chairpersons can be obtained from Exec. Director Walter Mientka, The University of Nebraska, 917 Oldfather Hall, Lincoln, NB 68588.

Mu Alpha Theta's first book of STUDENT PAPERS will be out very soon. It will be called, appropriately, "Mathematical Buds, Vol. 1". Papers for the second volume are now being solicited. A submitted paper, in order to be considered, must have won some top award in some competition, or, if there is none in the region, must have the sponsorship of a mathematics teacher. When submitting your paper, remember:

1. Title of paper
2. Name of author
3. Address of author, including the zip
4. Telephone number of the author, including the area code
5. The award the paper won, or the name of the teacher sponsor
6. Name of school attended and its address
7. Four (4) copies of the paper mailed to:

Harry D. Ruderman
2624 Davidson Avenue
Bronx, New York 10468

Include a self-addressed envelope to return the papers after the judging, and a self-addressed postcard to acknowledge the receipt of the paper.

In order to obtain possibly a more objective evaluation, the papers sent to the judges will have the author's name and address blocked out. If three of the four copies did not have this information, it would be very helpful for then it would not be necessary to block out.

The Nominating Committee is seeking candidates for Governors of Region III and IV and for President-Elect. Region III includes the states; Illinois, Indiana, Ohio, Pennsylvania, New Jersey, Michigan, Wisconsin, Connecticut, New York, New Hampshire, Massachusetts, Vermont, Maine, and any European countries. Region IV includes the states; Kentucky, West Virginia, Virginia, Delaware, Maryland, Tennessee, North Carolina, South Carolina, Georgia, Florida, Alabama, Mississippi, Cuba and any South American countries.

Nominations should be sent as soon as possible to:

Dr. Sarah Herriot
Mathematics Department
Henry Gunn Senior High School
Palo Alto, California 94306

INTRODUCTION:

The Four-Color Conjecture is now a Theorem, proven by mathematicians Kenneth Appel and Wolfgang Haken of the University of Illinois. Professor Appel was a speaker at the August, Mu Alpha Theta Convention in Dubuque, Iowa. At the 1976 summer meeting of the American Mathematical Society in Toronto, Professor Haken was a speaker and presented an outline of their proof. Stephen Wax, an eleven-year-old schoolboy, whose father is a microbiologist, attended this presentation with his mathematician grandfather, Dr. Goldberg. The following is his report to his teacher.

No One Sleeps at the Math Meeting

In a recent study, I have noticed that 1 out of 3 microbriologist fell asleep during a microbriology lecture. Yet, only one out 4,000 Mathmetician yawned!

Why is it that no one sleeps at the Math meetings but $\frac{1}{3}$ the Microbrologists fall asleep at their meetings, Is it the problems? The solutions. Is it the four color problem.

You may ask what the four color problem is. It's a simple problem but it's hard to prove. It's a problem where you have to prove that any map given, can be colored in four colors without any of the regions touching eath other with the same color. Mathmeticians have been working on this for 123 years. But finally the proof comes. Two guys spent 4 years and 1300 hours computer time (300 hours on a modern computer). They have $4\frac{1}{2}$ feet high of computer proof. Everyone is so exited about this. But who really cares, my box of crayons has 16 colors, so I don't really have a problem.

Giving a problem to a mathmetician is like telling a person to get the penny in the corner in the round room (It keeps 'em busy!)

A Mathmetician needs a problem like an alchoholick needs a drink! He needs a problem to be happy.

By,
Stephen Wax

CONVENTION HIGHLIGHTS

Approximately 325 persons attended the 1977 Mu Alpha Theta National Convention in Dubuque, Iowa, in August. Forty schools and twenty-five states were represented.

The winner of the Math Bowl competition was New Trier East High School of Winnetka, Illinois. Team members include Kei Mu Yi, Victor Milenkovic, Susan Levine, and Michael Spertus. The second place team in the Math Bowl was Gainesville High School of Gainesville, Florida, and the team members are Fred Tou, Philip Hren, Mark Bagnall, and Scott Blackowitz. Third place went to Wauwatosa West High School of Wauwatosa, Wisconsin, and the team members are Bradley Werner, Jay Nitschke, Mitchell Colton, and Alan Kopischke. In fourth place was Austin High School of Decatur, Alabama, and team members are Michael White, Richard Borie, Robert Sampson, and Jamie Oliver. Sponsors are Jeanne Travis of Gainesville, LeRoy Dalton of Wauwatosa, Gwen Snoddy and Thomas and Jeretta Thrasher of Decatur, and Sandra Whipple and Richard Rhoad of Winnetka.

An excellent program was provided. Loras College was a beautiful place and the people were great. Participants were both challenged intellectually and entertained royally. You missed a wonderful opportunity if you were not there.

A special thanks is in order to Miss Joyce Hubka of Wahlert High School in Dubuque who was Convention Chairperson and to Sister Paschal Nurré, the Program Chairperson. This list could extend to fill up The Log since so many people worked so hard. Let's just say, "Thanks Dubuque, Loras College, and Mu Alpha Theta members of Wahlert High School".

Plan NOW for next summer in Stevens Point, Wisconsin.

Attentive Listeners at Dr. Henry Pollak's presentation on "Mathematics and Industry" in Dubuque. (See---it pays off to sit down front.)

ARE YOU THINKING METRIC?

How many centimeters tall are you?
How many kilograms do you weigh?
How many kilometers is it from your house to your school?
How many square centimeters is the area of your desk top?
How many milliliters of uncola do you drink at lunch?
How many cheerliters do you have at your school?

(cont. from Pg. 1, col. 2)
an affair with Maxim Kowalesky who demanded that she give up her work to marry him. Finally, on February 10, 1891, she succumbed to influenza and died in Stockholm at the age of forty-one.

EDITOR'S NOTE: Dr. Grinstein has submitted biographical sketches of eight women whose contributions to mathematics have been notable. Other sketches will be presented in subsequent issues of The Mathematical Log. See if you can second-guess Grindstein and predict the other six. Big clues: They are all deceased; one taught in Bryn Mawr; See Dictionary of Scientific Biography.

U.S. MATHLETES WINNERS IN INTERNATIONAL MATHEMATICAL OLYMPIAD

A team of eight U.S. high school students won the first prize in the 19th International Mathematical Olympiad (I.M.O.) held in Belgrade, Yugoslavia on July 3,4. Two U.S. team members, Randall Dougherty of Fairfax, Virginia, and Michael Larsen of Lexington, Massachusetts, won individual first prizes with perfect scores on the two-day I.M.O. examination. Three U.S. teammates, Peter Shor of Mill Valley, California, Mark Kleiman of New York City, and Victor Milenkovic of Glencoe, Illinois, won individual second prizes. Team member James Propp of Great Neck, New York won a third prize.

The I.M.O. annually brings together teams of high school students from 17 nations for a spirited competition based on an examination requiring both broad knowledge and great mathematical ingenuity. The U.S. students topped teams from the U.S.S.R.(second place), Great Britain and Hungary (tied for third) and The Netherlands (fifth place).

The United States has competed in the I.M.O. only since 1974. This year's U.S. team is the first to win top honors, although the Americans have never finished below third place.

The U.S. team in the I.M.O. is chosen on the basis of performance in the U.S.A. Mathematical Olympiad which was held this year on May 3. The team was honored in Washington on June 7 at the Sixth U.S.A. Mathematical Olympiad Awards Ceremony and prepared for the I.M.O. at a training session held at the U.S. Military Academy, June 8-29.

The Mathematical Olympiad activities are sponsored by four national societies in the mathematical sciences. Financial support is provided by IBM, the Army Research Office, and the Office of Naval Research.

The Members of the U.S. team were:

Randall Dougherty, Fairfax, Virginia
Ronald Kaminsky, Albany, New York
Mark Kleiman, Staten Island, New York
Michael Larsen, Lexington, Massachusetts
Victor Milenkovic, Glencoe, Illinois
James Propp, Great Neck, New York
Peter Shor, Mill Valley, California
Paul Weiss, Brooklyn, New York

Sponsors of the U.S.A. Mathematical Olympiad and the U.S. team in the I.M.O. are:

The Mathematical Association of America
The National Council of Teachers of Mathematics
Mu Alpha Theta
The Casualty Actuarial Society

EDITOR'S NOTE: This is a News Release from The Mathematical Association of America. Dr. Nura Turner and Colonel Anthony Simkus provided us with the accompanying picture. An excellent write-up of the event appeared in the July 14, 1977, issue of the New York Times. Did you notice that Victor Milenkovic was on the Math Bowl team that won the competition in Dubuque, Iowa, at our Convention?

19th International Mathematical Olympiad

Members of the winning team shown in photo left to right are:
Randall Dougherty, Fairfax, VA; Mark Kleiman, Staten Island, NY; Victor Milenkovic, Glencoe, IL; Peter Shor, Mill Valley, CA; Ronald Kaminsky, Albany, NY; Michael Larsen, Lexington, MA; James Propp, Great Neck, NY; Paul Weiss, Brooklyn, NY; Dr. Murray S. Klamkin, Professor of Mathematics, University of Alberta, Canada.

LETTERS TO THE EDITOR

Flash! A Letter! Flash! A Letter! Flash! A Letter!

Dear Mathematical Log,

Here is some feedback on your CRYPTARITHMS, tax + tax + tax + tax = math; the answer is: 821 + 821 + 821 + 821 = 3,284. And now for my cryptarithm: ask · h.ow = me.ss

I also bring you some ABSOLUTELY |SILLY|s.

A person that can do anything. . . . . .Variable
Men that lie in the summer sun. . . . . Tangents
What the bird owner said to his parrot that was on a diet. . . . . .Polynomials
What advice would you give to a person who can't draw arcs well. . . . . .Triangles

I hope you like these as well as I like yours.

Your avid reader,
Todd D. Gatts

AN INTERESTING COMPUTATION

$$56^2 - 45^2 = 1111$$
$$556^2 - 445^2 = 111111$$
$$5556^2 - 4445^2 = 11111111$$
$$55556^2 - 44445^2 = 1111111111$$

* * * *

Can you prove that

$$(\underbrace{555...56}_{n\text{ digits}})^2 - (\underbrace{444...45}_{n\text{ digits}})^2 = \underbrace{111...11}_{2n\text{ digits}}\ ?$$

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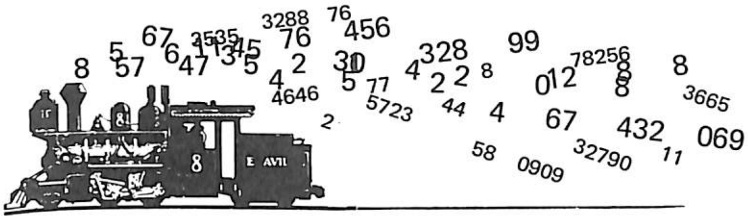

"Representation" Puzzlers Add Spice To Number-age Travel Adventures

Life, for all of us, is increasingly filled with numbers—and when we travel this can become particularly evident. Railroad rolling stock and highway buses, for example, almost always carry a numerical designation. These numbers, particularly four-digit numbers, can be thought of as posing an interesting travel-time challenge. One has just to consider them like this:

Taking the digits of the number in the given order and using (as required) the usual operations, signs, and conventions of elementary mathematics, what can be done to the value of the number; For example, with 9487, a coach on an overnight train through our town, the four digits can be used to represent a surprising variety of numbers. Consider:

$$9 + 4 + 87 = 100$$
$$(9 - \sqrt{4}) \times 8 + 7 = 63$$
$$9^{\sqrt{4}} - 8 \times 7 = 25$$
$$(9 \times 4 - 8)/7 = 4$$
$$9+4/(.8 - .7) = 49$$
$$9.4 + .8 - .7 = 3$$

In fact, starting with 1 (try it!), it appears that we can write **every** positive integer through at least 53, and 92 or more of the first 100 positive integers! We use a **9, a 4, an 8, and a 7,** in that order (let's agree) – with, as needed, signs of elementary mathematics including surds, exponents, parentheses, decimals, and repeating decimals . . . but not, we can agree, factorials, sigma notation, or other somewhat more advanced conventions. Order of operations (multiplication takes precedence over addition, for example) will need attention, as will proper use of parentheses. The challenges, given a (say) four-digit number, are (1) how high one can go, consecutively, using its digits (in order) to represent positive integers; (2) how many of (say) the first hundred numbers one can represent; and (3) how "elegant" a representation one can discover for some of the more challenging numbers.

In this spirit, see if you can better existing "records" for some particularly interesting "representations":

1. Awake through much of a memorable 23-hour train journey through Eastern Canada some years ago, I doodled the number of my "sleeping" car, 1439 (Canadian National, train 14, car 39). By journey's end, the four digits, used in order, had served to represent all numbers through 56 –also 90 of the first 100 positive integers.

2. "No. 5594," a Canadian National coach familiar in Maritime Provinces passenger services, offers a number with unusual possibilities. Through at least 86, all numbers can be represented. Indeed, 99 of the first 100 numbers can be set down if $.\bar{9}$, for "decimal 9 repeating" is allowed (and why not!) in writing 96. The Nova Scotia Teachers College Mathematics Club used my 5594 for its 1977-78 representation competition with most agreeable results.

3. Twenty years ago and more at the historic High School of Montreal, the young Mathematics Club I helped advise really enjoyed representaton problems, even ran them in the student newspaper. Now the school door number, 3449, itself presented interesting possibilities. Using **3, 4, 4, and 9**, numbers through at least 52 all can be represented, as can 96 of the first 100 positive integers.

4. Our coach on the overnight "Southern Crescent" from Atlanta to Washington one summer had the unpromising number, 3787. As it turns out, however, numbers through at least 25 all **can** be represented, as can 73 or more of the first 100 positive integers.

5. Riding Amtrak across the Garden State, we chanced upon Jersey Central locomotive 3681 outside Newark station. The locomotive number is a rather interesting one as 36, 81, 3 + 6, and 8+1 all

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"REPRESENTATION" (Continued)

denote squares and the final "1" allows for some interesting "decimal" representations – say (.3 + 6 - .8)/.1 for 55. Numbers through at least 30 all can be represented, as can 86 or more of the first 90 numbers. Seven numbers in the 90s, however, seem to cause trouble.

6. Take on if you will, the challenge of an essentially untried number, 6277, the Greyhound that carried a Mu Alpha Theta contingent from the convention at Stevens Point to Milwaukee. Note that 6, 2 (the first two digits) combine to at least ten values, in themselves useful building blocks for the more challenging representations.

These values are 2, that is, $\sqrt{6-2}$, 3, 4, 6, ($\sqrt{6^2}$), 8, 12, 27 (6/.2), 30, (6/.2), 36, and 62. Now, for a start, $1 = 6 + 2 - \sqrt{7 \times 7}$, $2 = 6/2 - 7/7$. . . !

"Equating" a seven-digit telephone number can be an interesting, if little known, variant on the foregoing. Mathematical expressions are constructed from the first three digits; also, from the final four digits. The usual challenge is to produce expressions that equate. Thus for 895-5347 (the College telephone number) one might write

$$8 + 9 - 5 = 5 \times 3 + 4 - 7,$$

since both expressions represent the same number, namely 12. A more worthy challenge, corresponding to a given telephone number, for how many different values can such a statement of equality be produced?

H.D. Allen
Nova Scotia Teachers College
Truro, Nova Scotia, Canada

SECRETARY'S CORNER

Math Fair Papers: Harry Ruderman reports that papers are still being accepted for the second booklet *Mathematical Buds.* For complete information check the Fall issue of *The Mathematical Log* or write Harry Ruderman, Hunter College High School, 94th & Park Ave, New York NY 10028.

Mu Alpha Theta is having a good year, 12,286 new members as of March 15, 49 new chapters chartered and 11 petitions for charters being processed.

Student Delegate Assembly: As you know, our National Convention is scheduled at Athens, Alabama, August 5-8. Last year we initiated a Student Delegate Assembly. Each Chapter sending a delegation should elect its representative and send the name and summer address to: Brad Long, 906 Wildwood Road SW, Decatur AL 35601. Brad is the student delegate for the Southeast Region.

A very successful Florida State Mu Alpha Theta Convention was held in February at Coral Springs High School. Nineteen chapters were represented by 427 students and teachers.

We're pleased to report Dr. Robert Wilson of Ohio Wesleyan University has been appointed to another three year term on our Governing Council. The appointment was made by Dr. Henry Alder, president of The Mathematical Association of America

Our two representatives on the High School Contest Committee, Richard Pieters and Martha Zelinka, report an excellent meeting in January with finalized work on the 1980 Examination and preliminary work on the 1981 Examination.

If you have any suggestions or comments on the Examination please send them to the National Office and we'll forward them to the Committee.

Each chapter should have received the following: *Mathematical Buds, Cryptarithms, Handbook for Sponsors,* and the article "The Charm of Mathematics" by Don Allen. If your Chapter didn't receive these publications, please contact the National Office.

NINTH NATIONAL CONVENTION OF MU ALPHA THETA – Aug. 5-8, 1979, Athens State College, Athens, Alabama

Plans for this year's convention are progressing well. Many schools from throughout the United States have expressed an interest and we hope that you are making plans to participate.

A detailed schedule of activities, information concerning the various competitions, regulations concerning conduct of students, etc., will be sent to those schools who send in the registration form.

Several questions have been asked about various matters. In response to these we make the following comments: (1) Students who graduate from high school

in May 1979 or later will be eligible to participate in the mathematics competition (2) The types of questions that will be used in the mathematics competition in each round will cover topics from elementary algebra through high school analysis (3) Students who would like to make presentations at the convention should notify us as soon as possible so that these may be worked into the schedule.

Please send us the student's name, school, topic, and a brief description of the presentation. Of course, several students may wish to cooperate on a presentation. If your students conduct such a session, we would appreciate your serving as the presider for their session. If you are willing to do this, please mention this when you send in the other information.

(4) If you have any other question regarding the convention, please let us hear from you. We're trying to plan the best possible convention. To receive a registration form, write immediately to Thomas N. Thrasher or Gwen Snoddy, Austin High School, Danville Road SW, Decatur AL 35601

ODE TO A MATHEMATICIAN

Thy symmetry hath my view ellipsed that thou hath transcendentalized mine own nature of values. Thy amplitude haste not to cross thy sine of happiness.

Though thou hath crossed my functions, mine own truth of the table shalt be.

Lori Calway
Ayer Jr. Sr. High School
Ayer, Massachusetts

ABSOLUTELY | SILLY |

Q. What do you do when Roxanne The Robot begins to squeak? **A. Euler** (it's not a bad joke if great care is taken to achieve proper pronunciation).

Q. M_CE, M_CE, M_CE. **A. Would you believe "Three Blind Mice"?**

And finally, here's one for all the seniors:

ur 2(nice) 2b 4 got 10

A LETTER FROM LYNNE

Lynne Hannah, tenth grade student at Crescenta Valley High School, La Crescenta, California, was first to decipher "Classic Beginning," the crossword-like grid featured in the Fall 1978 **Mathematical Log.**

Horizontal blocks of 5 squares correspond to letters in binary code – symbols being assigned in "what should be a familiar sequence," the order of letters on a typewriter keyboard. According to Dr. H.D. Allen, Nova Scotia Teachers College, who unveiled "Classic Beginning" at August's Stevens Point convention of Mu Alpha Theta, the quotation derives from the opening paragraph of Charles Dickens' classic, *A Tale of Two Cities* – a selection "interesting and perhaps unforgettable for its repetitive pattern of words".

According to Lynne, cracking "Classic Beginning" was "interesting and a lot of fun."

AND, A LETTER FROM LARRY

Dear Editor:

Recently, while looking at a chart of numbers and their squares, I noticed a pattern that exists among the numbers in the chart. I found that by adding to a number, the number just preceeding it, and the square of the preceding number, the resulting sum is equal to the square of the first number, hence:

$$2^2 = 4$$
$$3^2 = 9$$
$$3 + 2 + 4 = 9$$

After testing this pattern on several other numbers, I derived this formula:

$$n^2 = n + (n-1) + (n-1)^2 \text{ or } n^2 = (2n-1) + (n-1)^2$$

Larry Reid
Senior, Deshler High School
Mu Alpha Theta, President
Tuscumbia, Alabama

SUM OF SQUARES OF DIGITS

Let N be a positive integer (any number of digits). Define N_{k+1} = the sum of the squares of the digits in N_k for k = 1, 2, 3,...

Examples

a)

$N_0 = 16$
$N_1 = 1^2 + 6^2 = 37$
$N_2 = 3^2 + 7^2 = 58$
$N_3 = 5^2 + 8^2 = 89$
$N_4 = 145$
$N_5 = 42$
$N_6 = 20$
$N_7 = 4$
$N_8 = 16$ which repeats

b)

$N_0 = 86$
$N_1 = 64 + 36 = 100$
$N_2 = 1$ which repeats

c)

$N_0 = 1342$
$N_1 = 1^2 + 3^2 + 4^2 + 2^2 = 1+9+16+4 = 30$
$N_2 = 3^2 + 0^2 = 9$
$N_3 = 81$
$N_4 = 64 + 1 = 65$
$N_5 = 6^2 + 5^2 = 36 + 25 = 61$
$N_6 = 36 + 1 = 37$
$N_7 = 58$
$N_8 = 89$
$N_9 = 145$
$N_{10} = 42$
$N_{11} = 20$
$N_{12} = 4$
$N_{13} = 16$
$N_{14} = 37$ which repeats starting at N_6.

Investigate what happens for various starting values. After you have investigated, show mathematically that every positive starting-value, N_0, always converges either to 1 or to the cycle containing 4 which is given above.

Andree at Stevens Point

CONSTRUCTION WITH A SHORT RULER

Suppose we have an Euclidean ruler that is not a 'theoretical ruler' of infinite extent both ways, but a finite length of (say) 5 cm. Also suppose our compass cannot be spread more than 5 cm.

Using the above restrictions, prove that given any two points – no matter how far apart – we can draw the segment between them.

1) Given a 5 cm. ruler, we can connect any two points less than 5 cm. apart. Also by overlapping the ruler (see Ex. 1) on a previously drawn line, we can lay off a line greater than any given line. (This is a physical application of the Archimedean Property.) Now we are able to draw a line segment longer than any segment connecting two points, A and B, but we cannot guarantee that if the line passes through A it will pass through B.

2) How could we construct the perpendicular bisector of any segment? Given a line segment, we can use our compass to mark off equal lengths from each end. For example, given a line segment 15 cm. long we can use the compass to mark off 5cm. lengths from each endpoint (see Ex. 2). Now we can use our compass to construct a perpendicular bisector to the remaining 5 cm. segment. Since we have subtracted equal amounts from each end, the perpendicular bisector of the 5 cm. segment is also the perpendicular bisector of the original 15 cm. segment. Any line segment can be marked off repeatedly until the remaining center segment is 5 cm. or less, which will guarantee the construction of a perpendicular bisector for any segment.

3) Given a line l and a point P off the line we can construct a perpendicular line to the line from the point only if the point is closer than 5 cm. to the line. Our 5 cm. compass dictates this restriction (see Ex. 3).

4) We can, given a line l and a point P on l, construct a perpendicular to l at point P by using the usual construction methods. We can also construct parallel lines by constructing mutually perpendicular lines (see Ex. 4).

5) Given two points, A and B, construct the segment between them.

a) by using (1) we can construct a line through A and past B. Call the line I.

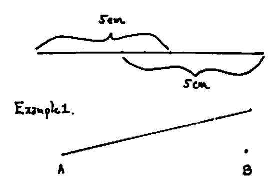
b) construct a perpendicular to I that passes near B as in (4). Call it K. K must be near enough to B such that we can use (3). If we are not 'near enough' on the first try, we try again (by the Archimedian Property we will eventually be 'near enough').

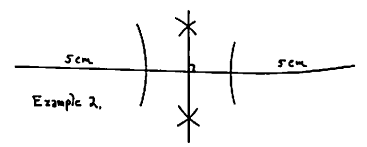
c) construct a perpendicular to K containing B, as in (3), call it L. Also construct a perpendicular to I at A. Call it M. Extend L and M until they intersect. Construct a perpendicular to L at B and extend it until it meets I. Call it N.

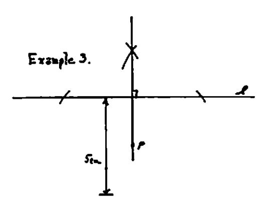
We now have a rectangle with A and B in opposite corners. We know from plane geometry that the midpoint of a line segment containing the midpoints of opposite sides of a rectangle is also on the diagonal of the rectangle.

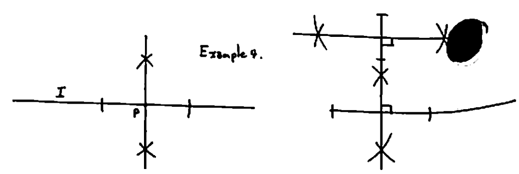
d) Use (2) to mark off segments of the side of the rectangle defined by line I and construct the perpendicular bisector. Extend it until it intersects line L. Use (2) to bisect this segment. Call the midpoint X.

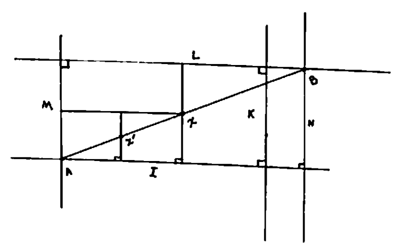
We now have a point X that lies between A and B. If the distance between A and X is still greater than 5 cm. we can repeat (d) using the rectangle with A and X as points in opposite corners. Similarly we can use (d) between X and B until we have enough points between A and B which are close enough to connect with a 5 cm. ruler.



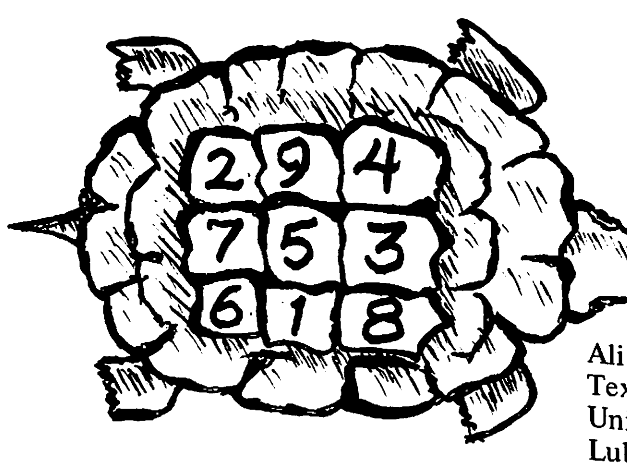








Cary Dachtyl, student
Richard A. Little, professor
Baldwin-Wallace College
Berea, Ohio



Ali R. Amir-Moez,
Texas Tech
University,
Lubbock, Texas

The Mathematical Log

VOLUME 25, NUMBER 1 -- OUR 25TH YEAR -- FALL 1980

UNDERGROUND MATHEMATICS

Tunnels and Tunneling

by Dagmar Henney
George Washington University

The first tunnel acknowledged in history was under the Euphrates more than 4000 years ago. It is incredible to think that the next tunnel under water was not until the Thames Tunnel in England in 1842. This earliest tunnel connected the Royal Palace with temples in Babylon. The early Greeks had one of their best known tunnels on the island of Samos in the Aegaen, described by Herodotus. It was started through a mountain from both sides of the mountain through limestone rock with hand hammers and chisels, with few irregularities though apparently not meeting at the end.

In the time of Augustus a tunnel was built for carriages to reach the favorite resort area of the Emperor in Baiae through rocky hills, connecting Naples and Posilipo.

The Hadrian Aquaduct supplied the ancient city of Athens with fresh water from Mount Parnes during the Roman occupation of Greece. It was discovered and rediscovered and finally renovated by Americans in 1972, and is now serving as an essential part of the Athens water supply.

Up to this time, tunnels were done by hand, drilling holes with hammers, chisels and wedges, then heating the rock face with fire and dashing cold water on it to cause fracture. In other instances, vinegar was used to break up the rock instead of water.

Gunpowder was introduced probably in the construction of a canal in France. Holes were filled with explosives and fired to break out the pieces. With this milestone, tunneling made great progress continuing to the railroad era. Convenience added to the necessity of creating tunnels for shipments in coal mining.

The first railroad tunnel was built in France (St. Etienne), and at this same time Brunel was struggling to complete the Thames Tunnel in England.

The Thames Tunnel project began in 1800 when there was no thought of shields. The essential idea of boring holes into the mountain and then lining that area was used. However, Brunel developed a cylindrical shield which consisted of a heavy steel plate fitting over the tunnel structure so that the front and back end could be wholly or partially closed. The shield was moved ahead by pressure of jacks pushing against the previously erected portion of the tunnel shell, like pushing a pole into mud. At the end of each forward movement the back portion of the shield still overlapped the end of the completed tunnel, providing an end within which the next ring of tunnel could be erected. Cast iron for lining was used at this time instead of brick. When completed, the Thames Tunnel was not used for 23 years. At long last it functioned as a railroad tunnel and was most useful.

Tunneling through the Alps was a great break-through in tunnels. The length alone of 8 mi. (13 km) through Mount Cenis was a challenge. Yet the ventilation was one of the greatest problems with the development of the steam engine. It seems ironic that when driving through a tunnel, people are most concerned about the countryside and scenery at the end of the tunnel, rather than the engineering which allowed them to pass in a few minutes through, say, 12 mi. (20 km) of mountainous terrain.

Subway tunnels were starting to be built in the 1800s. First the London subway system, the Paris Métro fashioned along the same lines as the London subway, Budapest, and then to America. The Moscow subway system, although built later, remains one of the most beautiful in the world--not a dark tunnel dusty and cold, but one which can be appreciated for its artwork of inlaid friezes lining the walls, colorful, massive, and eyecatching.

The uses of tunnels were wide and varied. Canals were being constructed in New Orleans for carrying flood waters; in Chicago under Lake Michigan to ensure safe

(Continued on page 3)

The official publication of the National High School and Junior College Mathematics Club, Mu Alpha Theta, which is sponsored by the Mathematical Association of America and the National Council of Teachers of Mathematics. Address correspndence to: Mu Alpha Theta, 601 Elm Avenue, Room 423, The University of Oklahoma, Norman OK 73019.

Secretary's Corner

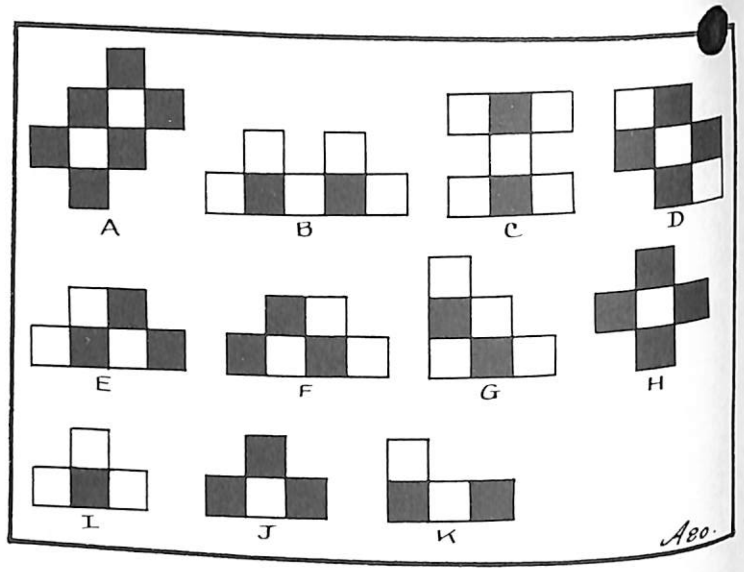
From Harold Huneke, Mu Alpha Theta Secretary-Treasurer, come the following items of information for chapters and individual members:

1979-80 YEAR "A GREAT ONE"

The past year was a great one for Mu Alpha Theta: 84 new chapters, 19 264 new members. Let's make 1980-81 even better.

INITIATION FEE CHANGE

Mu Alpha Theta chapters voted last Spring to change the one-time initiation fee (per member) from $1 to $2, the change to be effective August 1, 1980. A first-class mailing on this was sent out, but possibly we missed you.



ATLANTA CHALLENGE! The Editor's audiences at Mu Alpha Theta's 10th National Convention at Georgia Tech were left with this checkerboard challenge--an 11-piece dissection prepared especially for the event. The solution appears to be unique, and chapters are invited to share their findings. Reassemble the 11 pieces to form a perfect checkerboard, with alternating light and dark squares.

RETIRING EDITOR SAYS FAREWELL

Having completed her term as Editor of The Mathematical Log and taken on new duties within the National Council of Teachers of Mathematics, Dr. Betty Lichtenberg, University of South Florida, has penned the following as her farewell:

"It has been a great pleasure to be affiliated with Mu Alpha Theta and The Mathematical Log.for the past few years. "Contact with you all is one of the most rewarding experiences I've ever had. Thanks!

"Write letters to your new Editor!!"

--Betty Lichtenberg.

Sponsors Breakfast

A breakfast for Mu Alpha Theta sponsors will be held on Friday, April 24, 1981 in the Charles Lindberg Room, Sheraton Hotel, St. Louis, MO. The time will be 7 a.m. Reservations will be necessary, and should be made with the Mu Alpha Theta national office.

TUNNELS AND TUNNELING

continued from page 1

drinking water from the shoreline; also in Chicago, 60 mi. (100 km) of underground tunnels which few realize exist, carrying freight to the large department stores.

In the early 1900s, New York plants on Manhattan were becoming unable to cope with the rapidly growing demand for gas, and the congested area precluded the construction of large new plants. The alternative was a tunnel under the East River to pipe gas from Long Island into Manhattan.

When building the monumental Boulder Dam, rapid waters from the Colorado River were diverted for the period necessary for construction.

* * *

Methods used in tunnel construction have been, basically:

Open-cut method. A tunnel was constructed by excavating a trench to the desired depth, building a passageway in the trench, and refilling the space above the passageway.

Cut and cover. As pavement is taken up, usually at night, it is replaced by well-supported timber decks, under which the tunnel is constructed without disturbing life in the city.

Trench method. Used in tunneling under water. A trench of suitable size is dredged in the bed of a stream, and the bottom of the trench is properly leveled. Watertight sections of tube are then temporarily sealed at the ends by bulkheads which are floated into place and sunk to the bottom of the trench and the abutting ends of these sections joined together. Bulkheads or diaphragms are then finally removed to form a continuous passageway.

A lining is usually installed in a tunnel to prevent loose rock from falling, and is required to hold in place natural material surrounding the tunnel. The type and thickness of the lining will depend on the character of the rock.

* * *

Some tunneling records from the Guinness Book of Records:

Water supply tunnel: New York City-West Delaware, 1937-1945, 85 mi. (137 km).

Subway tunnel: London, 1939, 17 mi. (27 km).

Bridge-tunnel: Chesapeake Bay Bridge-Tunnel, Norfolk, Va., 1964, 17 mi. (27 km).

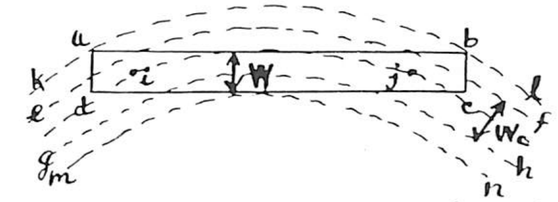
Canal tunnel: Rove Canal, between Marseille and Rhone River, 1912-1927, 4 mi. (6 km).

Railroad tunnel: Simplon II, Switzerland-Italy, through Alps, 12 mi. (19 km).

* * *

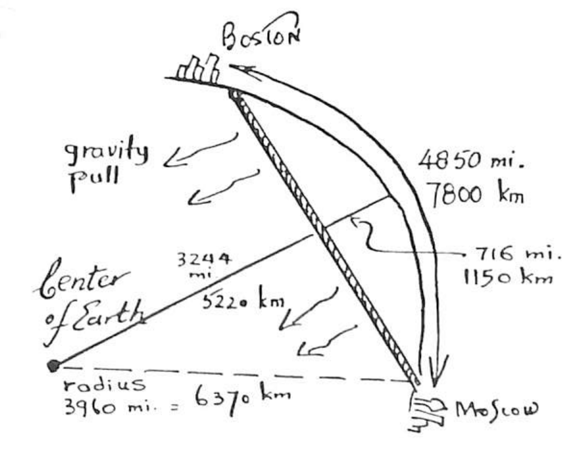
Several aspects of tunnel construction lend themselves to interesting mathematical treatment. Space permits development of but one.

When a railroad tunnel is on a curve, the width of the section must be greater than that of a straight track. Where a railroad car (abcd) is on a curved track whose rails are (ef) and (gh)--where the center is illustrated by (i) and (j); its ends project farther outside the track when it swings around a curve than when traveling on a straight track. The corners (a) and (b) travel along the curve at (kl). The central part of the car projects inside the track more than usual, therefore the path of the maximum projection is the curve (mn). In determining the path (mn) of the central part of the car, we must allow for tilting due to elevation of the outer rail. This elevation varies with the degree of curve and the higher speed of trains. The extra width of the tunnel on a curve must be at least equal to the difference between the required net width (W_c) and the width (W) of the car.



The distance between the outer rail (ef) and the path (kl) of the outside corners of the car is not the same distance as between the inner rail (gh) and the path (mn) of the inner face of the car. In order to keep the required width of tunnel as small as possible, a single curved track in a tunnel should be located so that the curve lying midway between the paths (kl) and (mn)--and not the center line of the track--will coincide with the center line of the tunnel. In the case of a double-track tunnel, the path (kl) for the outer track and the path (mn) for the inner track should be the same distance from the center line of the tunnel.

Reading into the history of tunnels will prove interesting, and the subject is rich in materials lending themselves to such mathematical interpretation. I want to conclude with the intriguing concept of a "global tunnel"--as once proposed by a free thinker named Cooper:



This tunnel (see diagram) would transport a passenger from any point on the earth in a stright line by means of a tunnel through the planet. The basis for "Cooper's theory" was that an object--a vehicle--dropped into a tunnel running (for instance) between Boston and Moscow would accelerate steadily during the first half of the trip to a speed which would produce the kinetic energy to "coast up" the other half against the pull of gravity . . . and reach any location in exactly 42 minutes!

dia Log ue
with the editor

DON ALLEN, THE LOG'S NEW EDITOR, HAS BEEN A FRIEND OF MU ALPHA THETA SINCE ITS EARLIEST DAYS. HE FOUNDED THE FIRST CANADIAN CHAPTER AT NORTHMOUNT HIGH SCHOOL, MONTREAL, IN 1960. MATHEMATICS CLUB SPONSOR AT NOVA SCOTIA TEACHERS COLLEGE SINCE 1969, DON WAS A PRINCIPAL SPEAKER AT MU ALPHA THETA'S 8TH NATIONAL CONVENTION IN STEVENS POINT, WI. DON'S HOBBY INTERESTS INCLUDE RECREATIONAL MATHEMATICS AND MONETARY HISTORY. HIS POSTGRADUATE DEGREES WERE EARNED AT UNIVERSITY OF SANTA CLARA, CALIFORNIA, AND RUTGERS UNIVERSITY, NEW JERSEY.

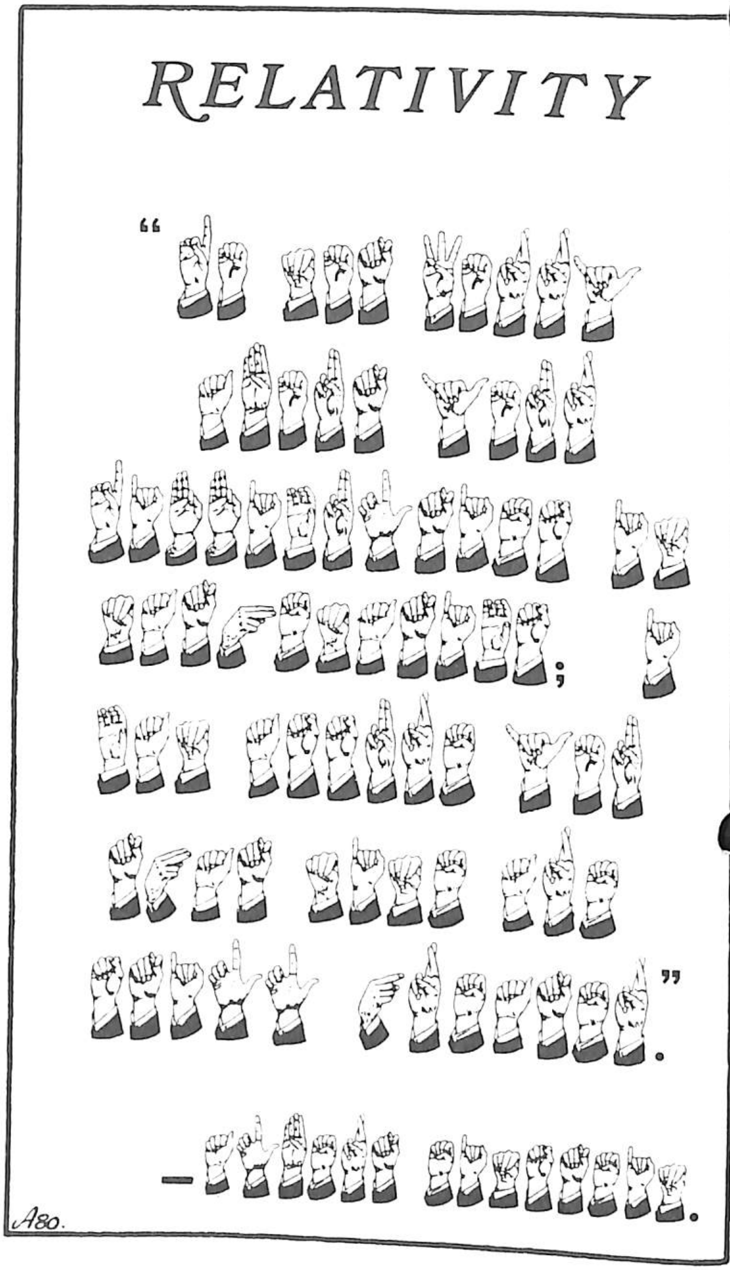
Our Georgia convention was outstanding! Pamela Drummond, general chairman, and Carol Wyndelts, program chairman, and Mu Alpha Theta students at Walton High School earned our greatest admiration for their efforts in planning and organization--and all came off exactly as planned. Now our sights are on California in 1981. At the Governing Council meeting in Atlanta, Fred Hansen offered an exciting preliminary report. The Log will have details next issue.

A special welcome, this first issue, to Mu Alpha Theta and to The Mathematical Log, to our perhaps 20 000 new members and to the thousands of others who are back with us for another year. The Log, to say the obvious, is your journal, it goes to every chapter. Use the Log. Share your good ideas for chapter programs. Tell your discoveries. Tell about worthwhile articles and books. Every chapter should be on the lookout for new ideas! Our own chapter opened its year with a presentation on the history of pi. Several "field trips" are in the planning stage.

Let's hear from you.

IRISH CATS DULY ANNIHILATED

"Annihilation," our transposition cypher in the Spring 1980 Log yields fairly readily once the significance of "gremlin," its 7-letter "codeword," is understood. One rewrites the "message" in rows of letters, 7 letters long. Resulting columns are headed with the letters of "gremlin," in alphabetical order: EGILMNR. Columns then are rearranged so that the letters spell GREMLIN. The thus reshuffled letters directly read off a charmingly nasty nursery classic, "There were two cats of Kilkenny . . . ." According to solver Lynne Hannah, La Crescenta, CA, this particular gremlin "took . . . a while to unravel."



RELATIVITY. The finger positions of "signing" comprise, in themselves, an elegant "code." Readers with a flair for ciphering may wish to direct their talents to the retrieval of a provocative quotation (above).

"MISSPELLED" NOT MISSPELLED

No, "misspelled" was not the misspelled word in "A Wit-Twisting Arithmetical Apptitude (sic) Test,"--"Apptitude" was, as numerous readers of the Winter 1980 Log were ready to point out. Those who wrote included Victoria Bychok, La Canada, CA; Holly Gwynn, Ft. Sta., VA; Laura Radic, Chicago, IL; Ray Stephens, Lenoir City, TN; Song Tan, Miami, FL.

The Mathematical Log
VOLUME XXIV, No. 2 — Winter 1980

The Birthday Problem Extended

A well-known probability experiment concerns the so-called "Birthday Problem": In a randomly chosen group of n persons, what is the probability that at least two persons share the same birthday anniversary?

To solve this problem, the probability that no two people share the same birthday is first caluclated. If the n persons were arranged in some order and each of them was to declare his/her birthday, then 365^n different n-tuples are possible (ignore February 29). If all birth dates are considered to be equally likely, each of these 365^n-tuples is equally likely as well.

To find the number of n-tuples which contain no matches, again visualize the n people declaring their birth dates in order. The first person may declare any of 365 days. To avoid a match, only 364 dates are available to the second person; 363 are available to the third person; 362 to the fourth person. The n^{th} person has (365-n+1) or 366-n dates available. The number of ways in which the n people can fail to have a match is (365)(364)(363)(362). . . (365-n+1). The probability of **no** matches is thus

$$\frac{(365)(364)(363)(362)\ldots(365-n+1)}{365^n}$$

The probability of **at least** one match is then

$$1-\left[\frac{(365)(364)(363)(362)\ldots(365-n+1)}{365^n}\right]$$

Surprisingly few people are needed in a group in order that the probability of at least one match exceeds 1/2. For 22 people, the probability of at least one match is

$$1-\left[\frac{(365)(364)(363)\ldots(344)}{365^{22}}\right] = 1 - .5243 = .4757;$$

however, the probability of at least one match for 23 people is

$$1-\left[\frac{(365)(364)(363)\ldots(343)}{365^{23}}\right] = 1 - .4927 = .5073$$

Let 23 be the critical value in the Birthday Problem: that is 23 is the smallest number of persons for which the probability of at least one match exceeds 1/2.

A similar experiment can be performed with decks of 52 regular playing cards. If each of a group of n-persons, draws a card from his/her deck, what is the probability that at least two of them will get exactly the same card (same suit, same value)? The same reasoning as that used in the Birthday problem shows that the probability of at least one match with n people is

$$1-\left[\frac{(52)(51)(50)\ldots(52-n+1)}{52^n}\right].$$

If n = 8, the probability of at least one match is .4325 while if n = 9, the probability of at least one match is .5193. The critical value for n is therefore 9.

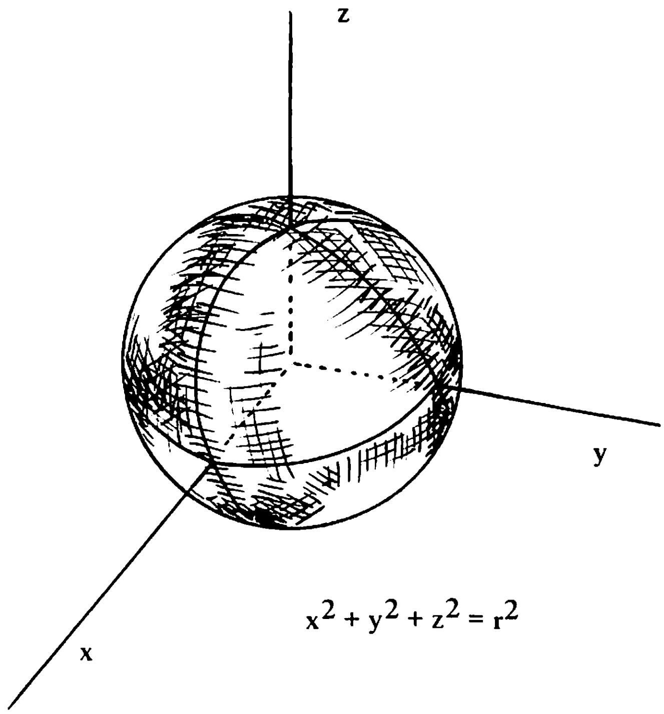
Let us generalize the problem. Suppose that each of n persons has a set of cards numbered consecutively from 1 to r. If each person selects a card at random from his/her deck, what is the probability of at least one match? The same reasoning as that used in the two previous examples leads to the conclusion that the probability of at least one match is

$$1-\left[\frac{r(r-1)(r-2)\ldots(r-n+1)}{r^n}\right].$$

For a given value of r, what is the critical number for n; that is, what is the smallest value for n so that the probability of at least one match exceeds 1/2? Table 1 depicts critical values for n.

(continued on page 4)

The Equation Of A Basketball



$x^2 + y^2 + z^2 = r^2$

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Editor: Dr. Betty Lichtenberg, Department of Math Education, University of South Florida, Tampa FL 33620.
MAA Representative: Prof. Robert Wilson, Math Department, Ohio Wesleyan University, Delaware OH 43015.
NCTM Representative: Alvin A. Gloor, 10925 Valley St., Omaha NB 68144.

Secretary's Corner

During the period from July 1, 1979 to January 15, 1980, 8,616 students have joined Mu Alpha Theta and 37 new chapters have been approved. Four charters are pending approval. We are alive and well.

We did not send the Instructor's Manual for *Cryptarithms* or *Logic Unlocks* to the chapters when we sent out the booklets. These manuals are available from the Mu Alpha Theta office. The cost to Mu Alpha Theta members is $1.20 each.

We regret to announce the death of Dr. Carl Olds, Professor-Emeritus of Mathematics at San Jose State University. Professor Olds was the first editor of *The Mathematical Log.*

Mu Alpha Theta is sponsoring a session on Mathematics Competitions at the Fourth International Congress on Mathematics Education to be held at Berkeley, California in August 1980.

State or regional meetings we know about are as follows. **Texas** — February 8-9 at San Antonio; **Tennessee** — March 5-8 at Smyrna; **Florida** — February 15-16 at Brandon; **Philadelphia** — May 23-24 — Regional; **Oklahoma** — October 14 at Norman. Would others planning meetings please notify our office so that we may publish your dates and locations?

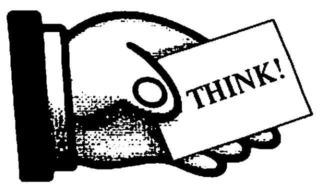
Students at Bronx High School of Science publish each year, *The Math Bulletin.* Their goal is to provide better communication in mathematical knowledge among students, parents and others. For information, write: Abby Amsterdam, Editor-in-chief, Math Department, The Bronx High School of Science, Bronx NY 10468.

Announcements

The annual breakfast for sponsors will be held at 7:30 a.m. Friday, April 18, at the National Council of Teachers of Mathematics meeting in Seattle, Washington. It will be held in the **Colonial Room** of the **Olympic Hotel** and the cost will be **$4.95 plus gratuity and tax.** Send reservations, no money, to Harold Huneke at the National Office.

Two other publications in the series developed by Richard and Josephine Andree are available from the Mu Alpha Theta office.

Secret Ciphers	$2.50
Secret Ciphers plus the Instructor's Manual	$3.50
Solving Ciphers	$2.50
Solving Ciphers plus the Instructor's Manual	$3.50



A WIT—TWISTING ARITHMETICAL APPTITUDE TEST

1. If you went to bed at 8:00 at night and set the alarm to get you up at 9:00 in the morning, how many hours of sleep would you get? ____
2. Do they have a Fourth of July in England?
3. Why can't a person living in Winston Salem, North Carolina, be buried west of the Mississippi River? ____
4. How many birthdays does the average man have? ____
5. If you have only one match and entered a room in which there were a kerosene lamp, an oil burner, and a wood stove, which would you light first? ____

6. Some months have 30 days and some have 31. How many have 28 days? ____
7. If a doctor gave you three pills and told you to take one every half hour, how long would they last? ____
8. A man builds a house with four sides to it and it is rectangular in shape. Each side has a southern exposure. A big bear wanders by. What color is the bear? ____

9. How far can a dog run into the woods that is seven miles in diameter? ____
10. What is the minimum number of active players on a baseball field during any part of an inning? ____
11. I have in my hand two United States coins which total 55¢ in value. Please bear in mind that one is not a nickel. What are the coins? ____
12. A farmer had 17 sheep. All but 9 died. How many did she have left? ____
13. Divide 30 by 1/2 and add 10. What is the answer? ____
14. Two people are playing checkers. They play five games and each person wins the same number of games. How do you figure this? ____
15. Take two apples from three apples and what do you have? ____

16. An archaeologist claimed he found some gold coins dated 46 B.C. Do you think he did and why? ____
17. A woman gave a beggar 50¢. The woman is the beggar's sister but the beggar is not the woman's brother. How can this be? ____
18. How many animals of each species did Moses take on the ark with him? ____
19. Is it legal in North Carolina for a man to marry his widow's sister? ____
20. If a plan crashed on the border of Mexico and the United States, where would they bury the survivors? ____

Bonus Question: What word is misspelled in this test? ____

Fill out your answers and send them to the Editor. We'll see the results in the next issue of The Mathematical Log.

1980 Convention Information

We hope that you're making plans to attend the Tenth National Convention of Mu Alpha Theta in Atlanta, Georgia, August 3-6. Several general sessions are being planned, including one for Sunday evening after which we will have the first round of the competition.

Some of the speakers for the general sessions will be: Dr. Jack Downes of Georgia State University, Dr. John Neff of Georgia Tech, and Dr. Tom Thomson of Kennesaw College.

In addition to the general sessions, there will be numerous section meetings, student presentations and the mathematics competition. On Tuesday afternoon, we'll go sightseeing in Atlanta on our way to Six Flags Over Georgia.

Each school that pre-registers will be sent a copy of the regulations concerning conduct at the convention.

Housing: On the campus of Georgia Institute of Technology, Atlanta, Georgia. Separate air-conditioned dormitories for girls and boys with two students assigned to a room. $1.00 refundable key deposit.

Costs: $51.00 per person for three nights lodging (linens included except pillows). Nine meals (Sunday night through Wednesday noon and Six Flags outing).

Pre-Registration Fees: $3.00 for members and sponsors, $5.00 for non-members.

For further information contact:
Pamela Drummond, Walton High School, 1590 Bill Murdock Rd., Marietta GA 30062. Phone: (school) 404/973-4250, (home) 404/255-2468.

The Birthday Problem Extended

(continued from page 1)

TABLE I

r (number of cards held by each person)	Critical value for n (Smallest number for which the probability of at least one match exceeds 1/2)
2	2
3 - 5	3
6 - 9	4 (Note r = 6 is the same situation as if each person rolled one hexahedral die and r=8, if each person rolled one octahedral die)
10 - 16	5 (r=12 is the same situation as if each person rolled one dodecahedral die)
17 - 23	6 (r=20 is the same situation as if each person rolled one icosahedral die)
24 - 32	7
33 - 42	8
43 - 54	9
55 - 68	10
69 - 82	11
83 - 99	12
100 - 116	13
117 - 134	14
135 - 156	15
157 - 178	16
179 - 201	17
202 - 226	18
227 - 252	19
253 - 280	20
281 - 309	21
310 - 340	22
341 - 372	23

As an example of how to read Table I, suppose that each person in a group draws a card at random from a set of cards numbered 1 through 250. Since, for r = 250, the critical value of n is 19, the probability of there being at least one match will exceed 1/2 if there are 19 or more persons in the group. If there are 18 or fewer persons in the group, the probability of at least one match is less than 1/2.

Challenges For The Reader:

1. Verify the critical values of n in Table I by direct calculation.
2. Extend Table I to at least r = 1000.
3. Table II reports the number of values of r associated with each critical value of n.

TABLE II

Critical Value of n	No. of Values of r
2	1
3	3
4	4
5	7
6	7
7	9
8	10
9	12
10	14
11	14
12	17
13	17
14	18
15	22
16	22
17	23
18	25
19	26
20	28
21	29
22	31
23	32

Conjecture the rate at which column 2 will increase using your results from Challenge 2.

Conjecture the rate at which column 2 will increase using your results from Challenge 2.

4. Program a computer to reproduce and extend Table I indefinitely with no need for operator input during the program.

David R. Duncan and Bonnie H. Litwiller
Professors of Mathematics
University of Northern Iowa
Cedar Falls, Iowa

MATH MIXT-MAXIMS

Make up more Mathematical Mini-maxims. The best ones we receive will be published in future issues of *The Mathematical Log*.

Answer: Pie In The Sky!

EAST CENTRAL JUNIOR COLLEGE
ESTABLISHED 1928
DECATUR, MISS. 39327

DR. C. V. WRIGHT, PRESIDENT
B. J. TUCKER, ACADEMIC DEAN
F. T. RIVES, REGISTRAR
C. D. BRACKEEN, DEAN OF STUDENTS
PERY B. WINEGARDEN, BUSINESS MANAGER

January 28, 1982

Dr. Maury Shurlds
Department of Mathematics
P. O. Drawer MA
Mississippi State, MS 39762

Dear Dr. Shurlds:

Mu Alpha Theta, the honors mathematics club at East Central Junior College, would like for someone from the mathematics department at Mississippi State University to speak at our meeting on February 11, 1982. The club will meet in Room 60, Newton Hall, at 10:35 A.M.

We hope that you can find the time to come visit us and give a short talk to these students on the needs and employment opportunities in mathematics.

Thank you for your concern in mathematics and our junior college students.

Sincerely,

Shelby L. Harris
Department of Mathematics

SLH:bd

Mu Alpha Theta Officers

Officers to the newly organized Mu Alpha Theta, an international junior college mathematics club at East Central Junior College are (left to right) Roger Gunn, president, son of Mr. and Mrs. Jimmy Gunn of Forest; Randy Russell, vice president, son of Mr. and Mrs. Clinton Russell of Decatur; Vikki Jenkins, secretary, daughter of Mr. and Mrs. William E. Jenkins of Philadelphia; and Terri Russell, treasurer, daughter of Mr. and Mrs. N. A. Russell of Decatur. Sponsor for the club is Dr. Shelby Harris (right), math instructor at East Central. The major purpose of Mu Alpha Theta is to stimulate a deeper and more effective interest in Mathematics.

HOUSE REPORT

By Rep. Eric Clark

Committees Staying Busy

Last week the state Legislature was working in high gear, with the members putting in long and busy days. Thursday, February 4, is the deadline for committees in the House of Representatives to pass general bills introduced in the House. The same is true in the Senate. So the different committees are meeting often, considering the many hundreds of bills introduced.

One of the committees which has been busy is the standing House Committee on Apportionment and Elections. I serve as secretary of this committee. We have voted out a number of significant bills in the past two weeks.

The most important were the new plans for "reapportioning" — redrawing the districts — for representa-

change in the state's Presidential election law. Under current law, the names of the seven "electors" pledged to vote for a particular candidate for President are all listed on the ballot, and the citizen must actually vote seven times. This is confusing and burdensome, and many voters do not take the trouble of checking the last names on the list.

The possible danger of this system was seen in 1980. The first of the electors pledged to Jimmy Carter almost got more votes in Mississippi than the seventh elector pledged to Ronald Reagan, though that clearly was not the desire of most Mississippi voters. House Bill 249 would allow our citizens to vote only once, for the "electors for" whichever candidate the voter is

A BIT OF COUNTY HISTORY!

Man Disappears, Then After Seeing 'Mysterio

By Sonny Renfroe

"My great-grandmother was known as 'Mammy' for miles around, and when anyone got sick, they sent for her. She was always willing to go and do what she could to relieve suffering, in fact she would have felt quite hurt if they had not sent for her.

One night a neighbor living about three miles away sent word for Mammy to come right over. My great-uncle saddled the mule, put Mammy on it and leading the mule, set out to take her to the neighbor's house. On the way, they passed Mt. Olive church, where they worshipped on the days when the preacher could get there.

It took no time to get to their destination, as the mule was fresh and to them the journey was nothing. My great-uncle helped Mammy off, hitched the mule and went in to see if there was anything he could do, and finding nothing, was soon on his way again.

As he came on down the road, this time riding the mule himself and whistling as he rode, he saw a light in the church which was dark

passed it the first time. He stopped, called out "Hello", as was the custom when hailing an unseen person. No answer. He called again. Still no answer. He got down, hitched the mule and went to the door. He saw a candle burning on the table, and no one to be seen in the church, which was only one small room. He called again, thinking that someone was camping there for the night, and had perhaps they had stepped out of the building for the moment. But he got no answer for the simple reason that there was no one there.

"He got on the mule and rode as hard as he could back to the sick neighbor's house, and tried to get some of the men to go back with him to investigate. But they just laughed at him, and told him he was nervous, and that he only thought he saw a light.

"He would not go back to the church alone, so he remained at the neighbor's house throughout the night, going back next morning with Mammy. For several days he had been planning to go for a visit with relatives in Madison County,


East Central


East
Central

THE MATHEMATICAL LOG

Volume X, No. 2 February, 1966

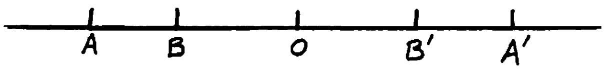
AN ELUCIDATION OF THE FOURTH DIMENSION

Many of you probably know that in mathematics the letter i represents $\sqrt{-1}$. If someone were to ask you to show him proof of the existence of i or to help him to conceptualize the square root of a negative number, you would probably hesitate. You know that a positive real number squared yields a positive real number, while a negative real number squared also yields a positive real number. The question arises, "What real numbers squared will yield negative real numbers?" Mathematicians, in order to solve equations such as $x^2 + 1 = 0$, extended the field of real numbers to the field of complex numbers.

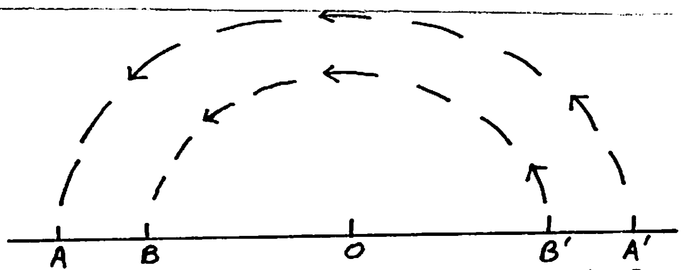
In a similar manner, although the layman may be unable to grasp the idea of a fourth dimension, the application of such a concept is fascinating and useful in the sense that it helps explain certain scientific phenomena —for example, Einstein's Theory of Relativity.

A) Before we explain the properties of the fourth dimension, it is necessary to understand the idea of symmetry and mirror images. An object is a reflection of another when they are equidistant from a point, line, or plane, and are congruent with respect to the point, line, or plane. The two objects are then said to be symmetric with respect to the point, line, or plane and are mirror images of each other.

Suppose we have a 1 D (one dimensional) world, a line m, and upon the line, two equal segments AB and A'B', equidistant from a point O as shown.

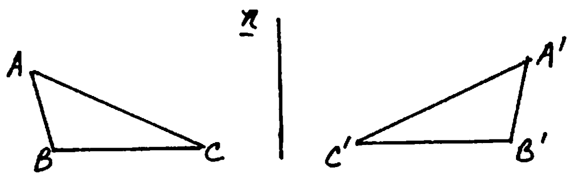


Remember that this line is a 1 D world and all movement is confined to one dimension. Now, removing AB and recording its exact position on the line, is it possible to coincide A'B' with the exact position that AB once occupied such that A' goes to the former position of A and B' goes to the former position of B? In a 1 D world this is impossible. However such a coincidence may be accomplished by rotating A'B' 18[illegible] in the second dimension.



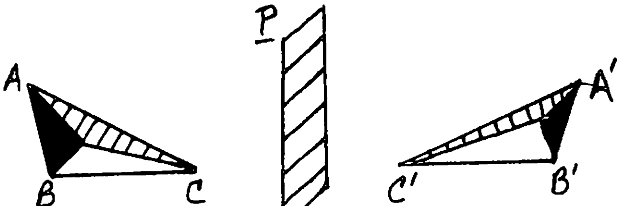
Now, AB and A'B' are symmetric with respect to the point O and are mirror images of each other.

Now, suppose we have a 2D world and two congruent scalene triangles equidistant from line n.



Is it possible, by manipulating triangle A'B'C' in only two dimensions, to coincide it with triangle ABC such that A', B', C' are superimposed over A, B, C respectively? This also is impossible. However, rotating triangle A'B'C' 180° around line n in the third dimension accomplishes such a coincidence. Thus, the triangles are said to be symmetric with respect to a line n and are mirror images of each other.

Finally, assume that in a 3 D world two congruent tetrahedrons are equidistant from a plane p, with no two faces of either tetrahedron congruent.



Continued on page 2

GLOSSOVER OF MATHEMATICAL TERMS

Sometimes we think of mathematics as a language, and indeed we often find a math problem essentially solved once we have figured out how to "say it in algebra," or "state it in mathematical terms." It is the language aspect of math that throws some students, too, especially those who are shy about risking faulty pronunciation or who "feel like fools" with new vocabulary. Maybe the following will help you feel at ease with some math words, but beware; some of our less responsible writers are not above joking, even on such a sedate topic.

sets—what mathematicians think makes the world go 'round
union of two sets—marriage
ellipse—monentary passage of lid across student's eye while teacher writes vital fact on blackboard
hexagon—square dance for witches
group—wheezy coughing of a German child
tangent—well-behaved male sunbather
graph—the fruits of crooked politics
non-parallels—little chocolate candies with white spots
paradox—two wharves
mathematical induction—initiation into Mu Alpha Theta
fulcrum—seedy drunk
binary—where the library book you need went
ternary—nesting place for gulls
irrational number—you must have been crazy to give that answer to Exercise 3
integer—the kind of *vitae* the Romans went in for before they switched to the *dolce* kind
homogeneous equations—the crazy mixed-up kind; hence by extension all equations
identity—state of having only one tooth
ind[illegible] locket pictures
[illegible] come
logarithm—woodsman's dance music
locus—nasty bug
nomogram—couldn't afford to have *initials put on it*
numerals—contemporary Post Office frescoes
prism—penitentiary
rhombus—what squares raise in a rhombus room—*I hat myself for starting this, but go ahead; send in yours. I deserve it.*

Margaret Maxfield

CHAPTER LETTERS

The Rancho High School chapter of Mu Alpha Theta, under the direction of Mr. Earl E. Hicks, Governing Council member from the Southwest Region, has had an actual demonstration of the Laser ray at one of their meetings. (LASER—light amplification by the stimulated emission of radiation).

The first two months of this school year, the National Office had received 22 Petitions for a charter in Mu Alpha Theta, and had enrolled 3,286 new members. It appears this year will be even busier than last, when 121 clubs were chartered and 13,386 new members were added to the rolls.

The Ryan chapter of Mu Alpha Theta sponsored a Mathematics Field Day last April for sixth, seventh, and eighth grade students. Leap frog, relays, chalk talk derbys, mad hatter marathons, nim and tic-tac-toe kept the participants busy.

If your chapter is interested in sponsoring such an event, write to Sister Mary Fred, O.S.F. Archbishop Ryan Memorial High School, Omaha, Nebraska, for information and materials they have prepared. Sounds like a wonderful project!

More than 400 members and advisers attended the eighth annual regional conference of Mu Alpha Theta Chapters in Ohio, Pennsylvania, and West Virginia held October 30, 1965, at Seneca Valley High School in Harmony, Pennsylvania. "Some Philosophical Aspects of Einstein's Theory," presented by Dr. Alax Thompson of the University of Pennsylvania, and "Mathematics of Space Exploration," presented by Mr. Myrl H. Ahrendt of NASA, carried out the conference's theme—Space Math—in the morning session. Representatives of five local colleges led the afternoon sessions entitled "Swales Construction for Determing Radius of a Given Circle," "Conversion of Numbers to Different Bases," "Role of Math in Science," "Roots of a Quadratic Equation," and "Sequence and Proof by Induction."

THE MATHEMATICAL LOG

February, 1966 **Volume X, No. 2**

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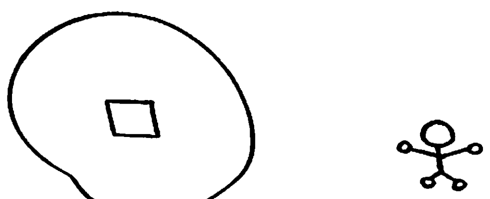
Continued from page 1

Analogous to our previous discussion, we can never make the two tetrahedrons coincide by any motion utilizing only the third dimension. However, the two tetrahedrons may be made to coincide by rotating either tetrahedron 180° around the plane p in the fourth dimension. The two tetrahedrons are symmetric with respect to a plane and are mirror images of each other.

Let us return to the 2 D world for another analogy. Assume there exists a 2 D being and a square completely enclosed by an unbroken circle.



The 2 D being, able to perceive in only two dimensions, is not able to see the interior of the circle since the circle acts as a boundary. Hence, the square enclosed by the circle is not visible to the 2 D being and is not within the reach of the 2 D being without his breaking the circle.

Nevertheless, a 3 D being, able to perceive in one dimension more than a 2 D being, is able to look into this 2 D world from his 3 D world and see the interior of the circle which the 2 D being is unable to do. Moreover, the 3 D being could pick up the square, move it in the 3 D world and replace it on the 2 D world outside of the circle. The 2 D being then sees what he thought was impossible—an object is moved out of the interior of a circle without breaking or even touching the circle.

In an analogous manner, suppose we have a 3 D world with a cube completely enclosed within an unbroken non-transparent sphere. A 3 D being cannot see the cube nor remove it without breaking the surface of the sphere. However, a 4 D being could easily see the interior of the sphere and could remove the cube without breaking the surface. Hence the strongest bank vaults would be no protection against the "super eyes" of 4 D beings!

B) Mathematicians may define the fourth dimension as a purely mathematical conception. For example, they could use the following system to illustrate what a 4 D space might be. The set of circles in a plane defines a 3 D space (two coordinates for the center, 1 for the radius (x,y,r)). Similarly, the set of spheres in 3 D spaces defines a 4 D space (three coordinates for the center, 1 for the radius (x,y,z,r)).

However, for this discussion we shall use a 4 D Cartesian coordinate system consisting of four mutually perpendicular lines intersecting at a common point called the origin (0,0,0,0) and a unit of distance. Some interesting properties are shown by analogies with lower dimensions.

In a 2 D space, the ordered pair (x,y) denotes a point in the 2 D Cartesian system. In a 3 D space, the ordered triple (x,y,z) denotes a point in the 3 D Cartesian system. Hence, in a 4 D space, the ordered four-tuple (x,y,z,t) denotes a point in the 4 D Cartesian system, where t is the distance along the 4th D axis.

In a 2 D space, a straight line is determined by the set of all ordered pairs (x,y) such that $ax + by + c = 0$. Using set notation, a straight line is determined by $\{(x,y) \mid ax + by + c = 0.\}$. In a 3 D space, a plane is determined by $\{(x,y,z) \mid ax + by + cz + d = 0\}$. Hence, in a 4 D space, a *hyperplane* (3 D space in a 4 D space) is determined by $\{(x,y,z,t) \mid ax + by + cz + dt + e = 0\}$.

In a 2 D space, the distance between two ordered pairs (x_1,y_1) and (x_2,y_2) is $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$. In a 3 D space, the distance between two ordered triples (x_1,y_1,z_1) and (x_2,y_2,z_2) is $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$. Hence, in a 4 D space, the distance between two ordered four-tuples (x_1,y_1,z_1,t_1) and (x_2,y_2,z_2,t_2) is $d = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2 + (z_1-z_2)^2 + (t_1-t_2)^2}$.

In a 2 D space, a circle is defined by $\{(x,y) \mid x^2 + y^2 = r^2$, where r is the radius of the circle$\}$. In a 3 D space, a sphere is defined by $\{(x,y,z) \mid x_2 + y_2 + z_2 = r_2$, where r is the radius of the sphere$\}$. Hence in a 4 D space, a 4 D *supersphere* is defined by $\{(x,y,z,t) \mid x^2 + y^2 + z^2 + t^2 = r^2$, where r is the radius of the 4 D supersphere$\}$. (For convenience the center of the circle, sphere, and supersphere is at the origin).

Suppose we have a 2 D space or a plane p. Let a straight line m on plane p be defined by $\{(x,y) \mid ax + by + c = 0\}$. The points (ordered pairs) of the plane not included on line m are defined by $\{(x,y) \mid ax + by + c<0\}$ and $\{(x,y) \mid ax + by + c>0\}$. The ordered pairs (x,y) satisfying the two inequalities represent two half-planes so that a straight line separates a plane into two components. Similarly, suppose we have a 3 D space. Let a plane p in this 3 D space be defined by $\{(x,y,z) \mid ax + by + cz + d = 0\}$. The points (ordered triples) not included on this plane are defined by $\{(x,y,z) \mid ax + by + cz + d<0\}$ and $\{(x,y,z) \mid ax + by + cz + d>0\}$ which in turn defines two half-3D spaces. Hence, a plane separates a 3 D space into two components. Finally, suppose we have a 4 D space. Let a 3 D hyperplane be defined by $\{(x,y,z,t) \mid ax + by + cz + dt + e = 0\}$. The points (ordered four-tuples) not included on this hyperplane are defined by $\{(x,y,z,t) \mid ax + by + cz + dt + e<0\}$ and $\{(x,y,z,t) \mid ax + by + cz + dt + e>0\}$, which in turn define two half-4D spaces. As expected a 3 D hyperplane separates a 4 D space into two components.

C) Another way to examine the fourth dimension would be to elaborate on the ordered four-tuples. Since $(x,y) + (a,b) = (x+a, y+b)$ in the system of ordered pairs, we analogously define the addition operation on four-tuples as

$$(x_1,y_1,z_1,t_1) + (x_2,y_2,z_2,t_2) = (x_1+x_2, y_1+y_2, z_1+z_2, t_1+t_2)$$

The real numbers are closed under addition (a real number added to a real number yields another real number). Therefore, the addition of ordered four-tuples of real numbers yields another four-tuple of real numbers.

The real numbers are also commutative under addition. (If A and B are any two real numbers, then $A + B = B + A$.) To show that the ordered four-tuples are commutative under addition, choose any two four-tuples (a,b,c,d) and (e,f,g,h). Then

$$(a,b,c,d) + (e,f,g,h) = (a+e, b+f, c+g, d+h)$$
$$\text{and} \quad (e,f,g,h) + (a,b,c,d) = (e+a, f+b, g+c, h+d)$$

But since a,b,c,d,e,f,g,h are real numbers, $a+e = e+a$, $b+f = f+b$, $c+g = g+c$, $d+h = h+d$. So addition of ordered four-tuples is commutative under addition.

The real numbers are associative under addition. (If A,B,C are any three real numbers, then $A + (B+C) = (A+B) + C$.) As a result, the ordered four-tuples are associative under addition. For any three ordered four-tuples (a,b,c,d), (e,f,g,h) and (i,j,k,l),
$(a,b,c,d) + [(e,f,g,h) + (i,j,k,l)] = [(a,b,c,d)+(e,f,g,h)] + (i,j,k,l)$ since $a+(e+i) = (a+e)+i$, $b+(f+j) = (b+f)+j$, $c+(g+k) = (c+g)+k$, $d+(h+l) = (d+h)+l$.

The ordered four-tuple $(0,0,0,0)$ is the identity element or zero element in this system as $(x,y,z,t) + (0,0,0,0) = (x,y,z,t)$ for any ordered four-tuple (x,y,z,t).

Each ordered four-tuple has an inverse. The inverse of any four-tuple (x,y,z,t) is another four-tuple which when added to (x,y,z,t) yields the identity element. Hence the inverse of (x,y,z,t) is $(-x,-y,-z,-t)$ since $(x,y,z,t) + (-x,-y,-z,-t) = (0,0,0,0)$.

Thus the system of ordered four-tuples under the specified definition of addition is closed under addition, is commutative and associative, has an identity element, and possesses an inverse for each element. By definition this system is an *Abelian group*.

Michael Takemori, Student
Elaine Tatham
University of Hawaii

BARGAIN

STUDENT-PRESENTED PROGRAMS

The students who gain the most from mathematics club experiences are the ones who present programs. If this statement is true, then it would seem that one way to have an effective mathematics club is to have a club in which the majority of its programs are presented by students.

The type of student-presented program that is recommended is one in which a student committee of from three to five students presents a mathematical topic by having each member of the committee present a subtopic. This type of program might vary in length from about one hour to one hour and a half. An example of such a student-presented program might be the following:

Topic: Groups

Possible subtopics:
1) definition of a group and a discussion of the group properties
2) permutation and symmetric groups
3) cyclic groups
4) the additive group of integers modulo n
5) commutative and non-commutative groups
6) finite and infinite groups
7) subgroups
8) elementary theorems about groups

Recommended references:

1) Carl B. Allendoerfer and Cletus O. Oakley, *Principles of Mathematics*, The McGraw-Hill Book Co., Inc., 1955, 1963.
2) Richard V. Andree, *Selections from Modern Abstract Algebra*, Holt, Rinehart and Winston Inc., 1958.
3) Richard E. Johnson, *First Course in Abstract Algebra*, Prentice-Hall, Inc., 1953.
4) John L. Kelley, *Introduction to Modern Algebra*, D. Van Nostrand Company, Inc., 1960.
5) M. Scott Norton, *Finite Mathematical Systems*, Webster Publishing Company, 1963.
6) John E. Yarnelle, *Finite Mathematical Structures*, D. C. Heath and Company, 1964.

All of the above subtopics would undoubtedly not be discussed at one meeting. It is only a suggested list from which the committe might select the subtopics that they would want to discuss. However, it is possible that all of these subtopics could be dealt with briefly. It is important that the committee coordinate their subtopics so that there is continuity and a minimum of overlapping of content. This can be accomplished through committee planning, sometimes with the sponsor and sometimes without the sponsor. The important thing here is that the sponsor knows at all times what is going on in the committee planning so that if his suggestions are needed to make the program more effective, he can give them at the times that they would be most effective.

The mechanics to use in planning of student-presented programs might consist of the following steps:

1) A calendar with meeting dates and times is set up for the year and is posted on the Mu Alpha Theta bulletin board.
2) The students are given about a week to sign up by committees for programs for which they will then be responsible. They are allowed to sign up with whom they wish. The more talented often sign up together; they are encouraged to present more difficult topics. If anyone fails to sign up for a program, the president of the club then signs them up.
3) The president calls a committee together approximately two weeks before the time at which they are to present their program. The sponsor meets with the committee and often the president does also.
4) If the committee has a topic in mind which seems reasonable, the sponsor can take them to the school library and suggest books for reference on the topic.
5) If the committee has nothing in mind for a program, the sponsor can tell them to browse in the library for a couple of days to see if they can find a topic that fascinates them. This serves to acquaint them with what is available in the library.
6) The sponsor meets with the committee again, and if they still have no topic in mind, he then suggests one.
7) With topic chosen, the committee is asked to read in general on the topic for awhile, using a variety of books.
8) The committee meets again and it is decided what subtopic each is to take and how long their talk is to be.
9) Each committee member then concentrates on study in his subtopic area.
10) Each member of the committee is encourage to practice his talk so as to be most effective in his presentation and to get a better concept of the time allotted for his part in the program.

The program time of one hour is divided equally among the members of the committee and if one or more go beyond their allotted time, the meeting time still generally falls within an hour and a half.

Often the committee arrives at a topic at the first meeting, in which case the number of planning meetings is cut down.

In the past four years, the following topics have been presented by student committees in the Wauwatosa West High School Chapter of Mu Alpha Theta in Wauwatosa, Wisconsin. A number of these topics have been presented more than once in this period of time by different committees.

mathematical paradoxes and fallacies
symbolic logic
topology
projective geometry
equidecomposability of polygons
magic squares
perfect rectangles
systems of numeration and their use in computers
non-Euclidean geometries
infinity and transfinite numbers
linear inequalities
vectors and vector analysis
probability
finite geometries
number theory
transcendental numbers
theory of relativity
convergent and divergent series
the lives and contributions of great mathematicians, such as:
Archimedes, Euclid, Descartos, Gauss, Newton
the abacus and rapid calculation
linear programming
matrices
game theory
cryptography, codes, and ciphers
mathematical games of hex and nim
life in an n-dimensional world
Boolean algebra and electrical networks
groups
~~*MAA Contest problems*~~
recreational mathematics problems and puzzles

To operate a club whose majority of programs are student-presented requires a mathematics library that contains at least four or five references for each topic that the club might wish to present. The library should contain up-to-date books, monographs, booklets, and journals. For a listing of up-to-date books for a mathematics club, it is suggested that the reader consult the Mu Alpha Theta Booklist. In the area of monographs and booklets, it is suggested that the library should include the SMSG's New Mathematical Library Series of monographs, a set of booklets on mathematical topics from each of the following organizations and companies: the National Council of Teachers of Mathematics, Addison-Wesley Publishing Company, Inc., Ginn and Company, D. C. Heath and Company, and the Webster Publishing Company. The journal list should include: "The Mathematics Teacher," "Scientific American," "University of Oklahoma Mathematics Letter," 'Scripta Mathematica," "Mathematics Student Journal," "Pi Mu Epsilon Journal," "School Science and Mathematics," "Mathematical Log," "Mathematics Magazine," and "American Mathematical Monthly." There should also be a number of college mathematics textbooks in the library for use by the more talented committees.

Student-presented programs can be successful for the following reasons:

1) Students like to be authorities on a subject. It gives them a feeling of importance to know more about a subject than their friends.
2) Students like to discuss, and sometimes even argue, mathematics and they seem to do more of it when students are presenting the program.
3) Student-presented programs aid attendance. When there are from three to five on the committee to present a program, they talk it up among their friends, while if there is to be a speaker, there is usually only one person actively involved, namely the one who is to introduce the speaker.
4) Students like to see their peers perform. They like to compare their mathematical abilities with those of their peers.
5) Students like to compete to see who can present the most interesting program.

Student-presented programs of the type described can be one answer to an effective mathematics club. If you have only tried this approach in a limited way, expand on it — give it a chance — it works.

LeRoy C. Dalton

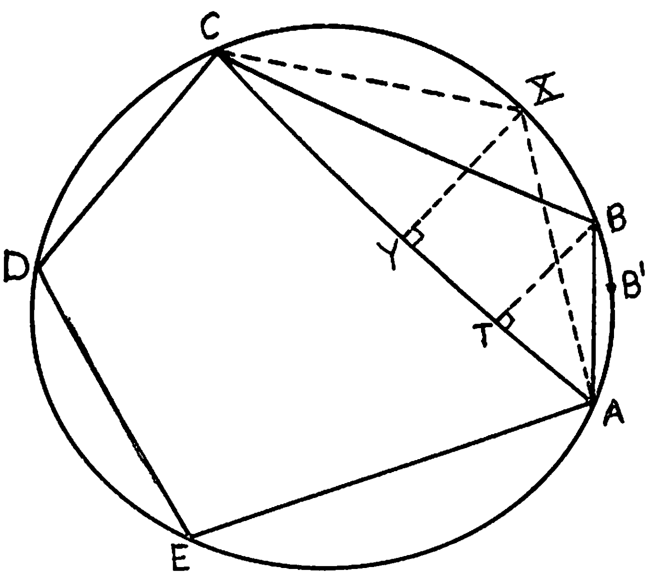
HOW BIG CAN IT BE?

Situations constantly arise in which we seek to make something as large as possible or as small as possible. For example, you have to return a book to the library, your mother wants you to get a quart of milk at the grocery, and of course you plan to stop and see your girl friend. How do you arrange the trip so that the least walking is involved? Or perhaps you plan to make a box to hold your school souvenirs. What shape should it be to hold them all but still require as little material as possible? Or possibly you have a certain fixed amount of money to use for recreation this month. How do you spend it to get the greatest number of activities (which is easy since you would just do the cheapest ones) or to get the greatest enjoyment (which isn't easy at all)? Or in buying a pair of shoes on your clothing allowance, what price should you pay to get the most months of wear per dollar?

Some questions of this kind are trivial while others are difficult and may demand mathematical tools such as perhaps calculus or linear programming which you have not yet considered. However, a surprising number, some quite complicated, can be treated by an intelligent use of facts which are familiar to all of us. The following is a problem of this type.

Let us consider a circle. If we select five points on the circumference of the circle they determine a pentagon, as shown in the figure. The problem is to find the largest such pentagon, that is, the one having the largest possible area. Can you make a guess as to what this best possible shape should be? (Intelligent guessing is a very important asset in mathematics and is discouraged only when it is used as a substitute for proof). Would you guess that the best pentagon would have the sides all of equal length? It certainly seems a fairly plausible conjecture. Let us see what progress we can make toward discovering whether or not it is actually correct.

Let ABCDE as shown be any pentagon inscribed in the circle for which the sides are not all of the same length. Then there must be a pair of adjacent sides of different lengths. To be definite suppose that AB is shorter than BC so that B is not the midpoint of the arc ABC. Let us draw the chord AC dividing the pentagon into quadrilateral ACDE and triangle ABC. Let X be the midpoint of arc ABC and compare triangles ABC and AXC. They have a common base AC but the altitude XY of AXC is clearly greater than the altitude BT of ABC since the midpoint of an arc has the greatest distance from its chord. But this means that triangle AXC has greater area than triangle ABC and therefore that pentagon *AXCDE has greater area than the given pentagon ABCDE, so the given pentagon, that is the inscribed pentagon with all sides equal in length.*



This discussion shows that no pentagon having two different lengths for its sides can be the biggest one since we can always find a larger one. Hence the only possible candidate for a largest pentagon is the regular pentagon, that is the inscribed pentagon with all sides equal.

At first sight it seems that we have completely established that the regular pentagon has maximum area but this is not quite true. What we have actually showed is that no *other* pentagon *can* have maximum area. Of course if someone will assure us that there *is* a pentagon with maximum area then we will be sure it must be the regular pentagon since all other possibilities have been eliminated. However, as matters now stand we are not sure that there is a pentagon of maximum area at all. Actually this fact can be proved and the above discussion then does show that the regular pentagon has maximum area. The proof that there really is a biggest pentagon involves some complications that it seems better not to discuss here however

It is worth noticing that the question of whether there is or is not a figure satisfying a certain description is not just a matter of hairsplitting. For instance suppose in our discussion we ask about the inscribed pentagon of *smallest* possible area. In the figure let ABCDE represent *any* inscribed pentagon. Then we see that by replacing B by some other suitable point, say B′, the triangle AB′C will have smaller area than the triangle ABC, so that pentagon AB′CDE has smaller area than the given one ABCDE. According to this, no matter what pentagon we are given there is always a smaller one, so there clearly cannot be a smallest one.

You may find it amusing to verify that the same argument can be applied to inscribed figures with any given number of sides. Thus you can show that the equilateral triangle is the largest inscribed triangle, the square is the largest inscribed quadrilateral, and so on.

Stanley B. Jackson
University of Maryland

NOW is the time for seniors to apply for admission and scholarships at the University of their choice.

THREE MATHEMATICAL TABLES YOUR LIBRARY SHOULD HAVE

Every high school library should have some recent books of mathematical tables. Here are three volumes which I recommend to your librarian.

1. A standard set of matematical tables is published by the Chemical Rubber Company and is now in its fourteenth edition. It is available in several different editions. Your author's favorite is the *Handbook of Mathematical Tables,* 2nd Edition, Catalog Number 625 which costs $7.50. Similar material, although in smaller format, is available in the *CRC's Standard Mathematical Tables,* 14th Edition, Catalog Number 624, at $5.50, or in the Student Edition, Catalog Number 614, at $4.25. The larger format of the $7.50 volume is worth the difference in the opinion of this reviewer. The tables are available from the Chemical Rubber Company, 2310 Superior Avenue, Cleveland, Ohio, 44114.
2. A second set of tables which we recommend, is the new *Handbook of Mathematical Functions,* published by the National Bureau of Standards. The Catalog Number is 55, and the price is $6.50. It is available from the Superintendent of Documents, United States Government Printing Office, Washington, D.C., 20402, with a paperback edition available from Dover Publications, 180 Varica Street, New York, 10014, at $4.00.
3. A third set of tables which we feel should be in every high school library is the *Tables of Indices and Power Residues,* published by the W. W. Norton Company, 55 Fifth Avenue, New York, New York, 10003, at $10.00. This set of tables of the most important function in number theory has an introduction by H. S. Vandiver that provides an excellent historical introduction to some of the major problems of the theory of numbers.

If your library is missing any or all of these mathematical tables, by all means encourage the librarian to order them as soon as possible. It is quite probable that the current printing of tables two and three mentioned above will be exhausted soon. The demand has been appreciably heavier than was originally anticipated, so send your order in now.

R. V. Andree

MATHEMATICAL LOG PROBLEMS

1. A man dropped a bottle over the side of a boat while rowing upstream. He continued to row upstream with the same steady expenditure of effort for 3 minutes after dropping the bottle. He then turned the boat around and rowed downstream for a distance of two miles at which point he recovered the bottle (two miles relative to the bank of the river from the point where he turned the boat around to the point at which he recovered the bottle). How fast was the man traveling (relative to the bank of the river) as he was rowing downstream?

2. Two positive numbers differ by 25. Their arithmetic mean exceeds their geometric mean by 5. Find the numbers.

3. The diagonal of one square is $a + b$. Find the perimeter of a second square which has twice the area of the first square.

4. Describe the subset of xy-plane which is given by $\{(x,y) : |x| + |y| \leq 1\}$.

L. A. Ringenberg
Eastern Illinois University
Problem Editor

THE MATHEMATICAL LOG

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THE TESSERACT

Not many mathematical concepts catch the popular imagination quite like that of the fourth dimension. However most of us have no experience in visualizing or sensing or even trying to visualize or sense more than three dimensions. In fact, it has been said that the human mind cannot conceive of more than three dimensions.

Our experience with coordinate geometry compels us to think of two dimensions in terms of two perpendicular lines and to think of three dimensions in terms of three mutually perpendicular lines. But can we think of four mutually perpendicular lines?

We are going to try to develop a visualization of at least one four dimensional object, the four dimensional cube or tesseract. There are many, many other four dimensional objects and we will touch on them lightly. To help yourself visualize these strange objects, prepare to allow your imagination free rein.

Before we get too far into the wilderness of the fourth dimension, let us get our footing and review some of what we know about the lower dimensions. As you probably know, the usual Cartesian coordinate system gives a good mathematical representation of two dimensional space using two perpendicular lines (see Figure 1).

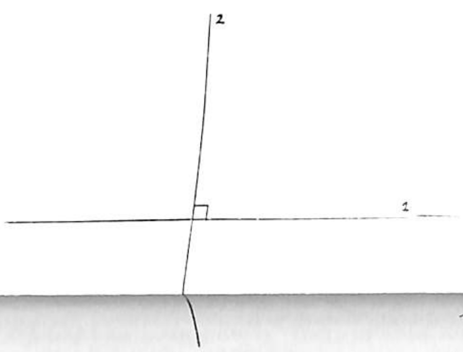


Figure 1

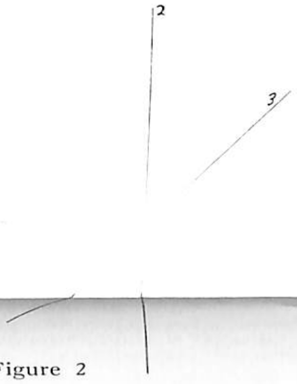


Figure 2

In order to represent three dimensional space, we start with the two perpendicular lines of two-space and add a third perpendicular to both (see Figure 2). When we try to draw this on a flat sheet of paper or a blackboard, as in Figure 2, we draw the two perpendicular lines of Figure 1 and draw the third line as if it bisected the first and third quadrants. We visualize this line as coming out of the paper towards us; in fact, the portion in the third quadrant is to come up out of the paper and the portion of the first quadrant is to be thought of as if it were behind the paper.

As the preceding discussion shows, we have to use our imagination to visualize a three dimensional set of axis as drawn on paper; we have to have a sense of perspective. This is basically our problem in visualizing a four dimensional object using only a two dimensional page of paper. This is akin to describing a cube by looking at its projection on a straight line. It will call for a lot of imagination! For these reasons, the reader is vigorously encouraged to make models from pipe cleaners or matches so that the visualization can be helped by using three dimensions.

One way to draw a cube is by first drawing a square (see Figure 3a) on the paper; then draw a second square oriented the same way and overlapping the first (see Figure 3b). Now try to visualize the second

Figure 3a

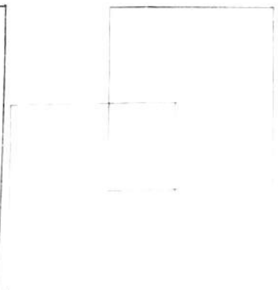


Figure 3b

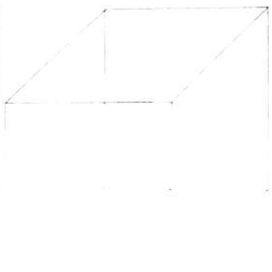


Figure 3c

square in a different plane from the first; in a plane above the first. Finally, draw lines joining the corresponding vertices as in Figure 3c, and we have a representation of a cube.

Now continue another step. Take the cube we now have drawn and draw another cube of the same size, superimposed on the first and overlapping it and oriented the same way (see Figure 4a). Now join the corresponding vertices. Each cube has eight vertices so there are eight

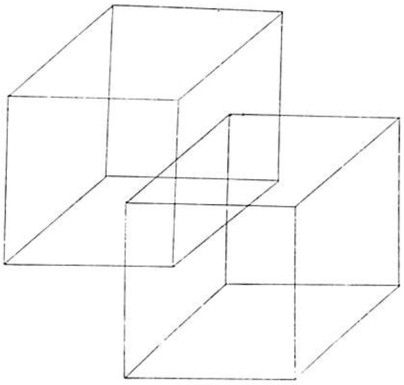


Figure 4a

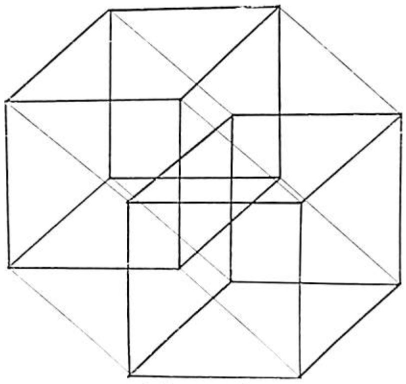


Figure 4b

joins (see Figure 4b). This resulting figure is a representation of a four dimensional cube or tesseract as drawn on a plane.

We could continue this process another step and try to draw the fifth dimensional cube, but the figure is very, very complicated.

It may help to get a different perspective on the tesseract. First draw a cube and another inside it oriented the same way (see Figure 5a).

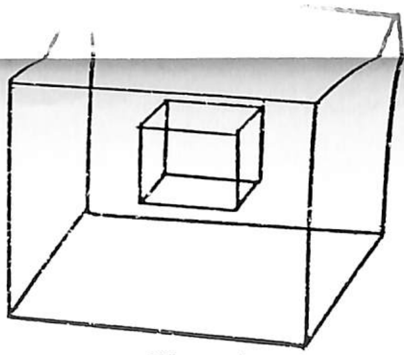
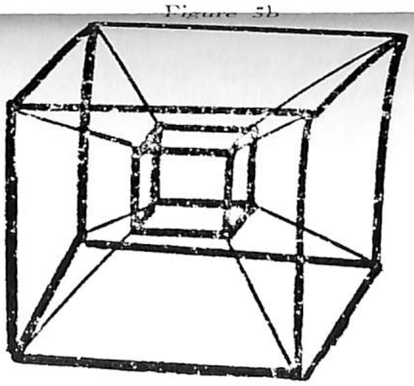


Figure 5a

Figure 5b



Now join the corresponding vertices as in Figure 5b. This view of the

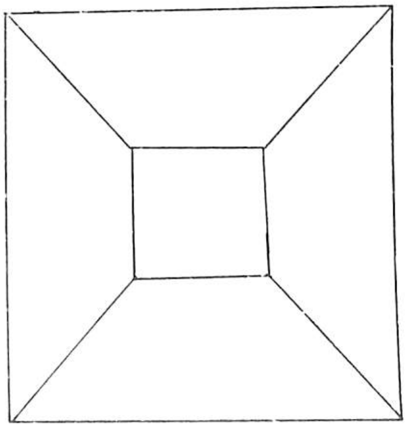


Figure 6

tesseract is analogous to looking into a box from an open end with your face close to the opening (see Figure 6).

(Continued on page 4)

THE MATHEMATICAL LOG

December, 1973 Volume XVIII, No. 2

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NATIONAL CONVENTION

Plans are progressing for the next national convention on August 4-7, 1974, at the University of Arkansas. Your chapter should have received a mailing on the meeting recently. If not, write to the general chairman, Mrs. Marian Crum, Fayetteville High School, Arkansas 72702.

SECRETARY'S CORNER

For several years Balfour has had available two qualities of Mu Alpha Theta insignia, the 10k gold and the Balfour golden finish. The fluctuating price of gold has made it difficult to maintain a stable price on the 10k gold insignia even for one year, and it seems this pattern will continue. They have also developed a new gold finish, "Balclad," which they feel is better than the present Balfour golden finish. In view of the above, *the Governing Council of Mu Alpha Theta has voted to discontinue the 10k gold and Balfour golden finish insignia when the present stocks are depleted and to then have only the Balclad finish insignia.* The prices on the Balclad insignia are the same as for the present Balfour golden finish.

We are now working on a printing of a revised roster of Mu Alpha Theta clubs and plan to send a copy to each chapter along with the April *Log*.

NEW FRIENDS

Wear your Mu Alpha Theta pin when you attend meetings, conferences, and other events away from your school. It will help you to be identified by strangers with similar interests. Be on the watch for Mu Alpha Theta pins yourself, and extend a cordial greeting to other club members.

CLUB NEWS

The Hubbard High School club in Chicago is holding a Mathematics Game Fair in December. This will involve the student body in recreational mathematics including student-oriented mathematical games and computer games. In the spring they plan to sponsor an Eighth Grade Mathematics Contest with student-written questions using the format of a local TV quiz show.

The chapter of St. John's High School in West Islip, New York, sponsored a successful regional meeting this fall with Dr. Julius Hlavaty as the featured speaker.

We need your help in reporting special activities of your club. Through *The Log* you can share your idea with others.

Several clubs are raising money for the Fayetteville, Arkansas Mu Alpha Theta meetings in August 1974 by selling sets of reprints of mathematical articles from the *Log* and the *O.U. Math Letter* at $5 per set of five different booklets. Chapters may order 10 sets (50 books) for $22.50 and make $27.50 profit on each ten sets sold. ACT NOW.

BREAKFAST FOR SPONSORS

Plan now to attend the breakfast for sponsors at the NCTM convention in Atlantic City. The breakfast will be at the LaFayette Motor Inn on Friday, April 19, 1974, at 7:30 a.m. They serve a buffet breakfast for $2.50 and will provide us with a meeting room. Reservations can be made by writing to Dr. Huneke at the national office. You may pay for the reservation at the breakfast.

PROBLEM SOLVING for fun and profit

We are interested in all types of problems: problems that involve mathematics, politics, pollution, teaching, government, sociology, psychology, and the problem of getting some particular thing accomplished by your community or school. It may never have occurred to you that problems of physics, chemistry, sociology, politics, psychology, mathematics, biology, and social science have much in common—but each requires a combination of orderly thinking and luck to derive acceptable solutions. We can't do much about increasing your luck, but have observed that good fortune favors the person who is well prepared. We shall suggest ways in which *you* can attack problems and organize your thinking in ways experts have found helpful.

If we are going to learn to solve problems, it may be best to start with problems that are easily understood. Just because a problem is easily understood in no way implies that it is easy, or even possible, to produce a solution to the problem. Our first set contains problems from plane geometry and from the very ancient and honorable discipline of integer (whole number) arithmetic with which every reader is presumably familiar. Euclid too worked in both areas.

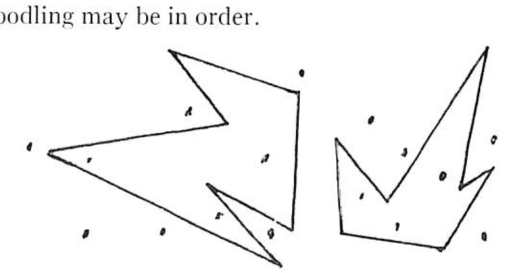
The reader may be surprised to discover that in many "true life problems," the most difficult part is to obtain a good *clear statement of what the problem is* and of *what restrictions are to be placed on the acceptable methods of solution*—but it is true. If your objective is to knock down as many bowling pins with two balls as is possible, then one solution might be to tie the two balls together with a rope and "gutter" the two balls simultaneously, but somehow that is not an acceptable solution in most bowling leagues. Similarly, many social and economic problems have valid solutions which violate the "acceptability criteria" of our society at this point in time. First, we shall discuss mathematical problems that are well defined and easy to understand, even if not always easy to solve.

Problem 1

Given a simple (non-intersecting) closed finite plane polygon G (not necessarily regular or convex). Then given any point P in the plane which is *not on the perimeter of the polygon*, is it true or false that at *least one entire side (line segment) of polygon G must be visible from the point P?*

This seems a reasonably nice problem in that, with the possible exception of the word, "visible," the meaning is clear. If you have any doubt what "visible" means in this context, create a mathematical definition of your own choice that will also satisfy two of your colleagues and use it.

A bit of doodling may be in order.



In the two polygons given above, it is certainly true that from any of the indicated points, or from any other point you choose there is always at least one entire side of the polygon that is visible from the point.

Of course, that does *not prove* that this is always so. Try your hand at a few polygons of your own choice.

It does seem reasonable that the proposition might be true, does it not? For good and valid reasons mathematics does not accept "proof" of its "laws" by lack of counterexample. Until one either has a proof or a counterexample, one uses the term, "conjecture," to describe a statement believed to be true, but still unproved. We now have a Conjecture (to be proved or disproved)

Given a simple closed finite plane polygon G, and given any point P not on the perimeter of G, then at least one entire side of G is visible from the point P.

The conjecture is now yours to consider. See what you can do on producing either a proof or a counterexample (or both??).

Prime integers have been investigated for a long time, but still preserve their secrets well. It is interesting to think of functions P(N) such

(Continued on page 4)

COMPUTER MUSIC

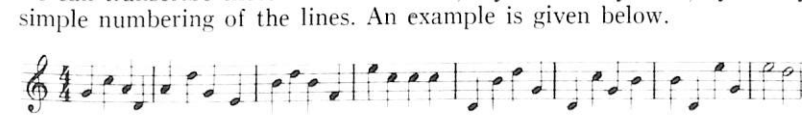
If one accepts the general definition of music as "modulated sound that produces (usually) emotionally pleasurable sensations in the listener," then music and computers had an early marriage. Many of the early computers had a loudspeaker hooked into the circuits. The resulting series of squawks and squeals clued the experienced operators to the fact that the computer was in a loop. Operators became familiar with the expected sound patterns of frequently used programs and could often spot trouble by auditory monitoring of the circuits. You may obtain a similar effect by placing a transistor radio on your computer and tuning it a bit. It is a short step from auditory monitoring to deliberately having the computer produce modulated sound that resembles "Coming Round the Mountain" or "Oklahoma" or "Frere Jacques" or some other simple tune. This does not, in your authors' opinion, have any real significance as far as music is concerned and probably should not be indulged in, since it gives the uninitiated the wrong idea about what computers do. A more sophisticated approach has been taken in which an actual study of harmonics involved in various instruments leads to the use of computers in storing magnetic spots on a piece of magnetic tape (a common computer storage device); these magnetic tapes are then played in a tape recorder. An understanding of the basic identifiable sounds of a given instrument has made it possible to use the computer to produce sounds which are clearly recognizable to the skilled ear as having come from a flute or a piano or any of a number of other instruments. Theoretically, it is even possible to combine these to produce the entire sound of an orchestra, although to my knowledge, this has never been done. Actually, while interesting, this too is not really important. If you want to have a tape recording which sounds very much like a good flute being played, the thing to do is to engage, for an hour, the first flutist of one of the major symphonic orchestras and make a tape recording. It will be every bit as satisfactory as anything you can produce on the computer and much cheaper. It is also possible to analyze the harmonics and overtones without constructive simulation.

Actually, the computer does have an important role in modern music. It is an *instrument* which will open entirely new and different doors to the musician of the future. When a musician plays on an ordinary instrument there are limitations imposed by the instrument itself. For example, there is a certain minimum tone duration. No note may be sounded on that instrument for a time shorter than its minimum duration. It is also quite possible that there are certain notes which cannot be sounded simultaneously. On a piano, for example, it is very difficult to play a high "A," a middle "C," and a low "B" unless one uses one's nose. Similarly, there are certain beats and vibrations which are so difficult to achieve that musicians have considered them to be impossible on ordinary instruments. The computer is not subject to these limitations. Almost limitless sequences and shadings of sound may be produced. The student who is interested may hear some of the early efforts on Decca Recording DL 9103, entitled "Music from Mathematics" which was actually played by an IBM computer using the Bell Laboratories Digital to Sound Transducer.

There are several programs which use the computer to actually *compose music*. The purpose of such projects is often misunderstood. It is quite possible to play a sequence of pseudo-random notes on some instrument, and call the resulting output music. For example, if we use a table of one-digit pseudo-random numbers (the first line of which is reproduced below)

5 8 6 2 6 0 5 3 7 9 7 4 0 8 8 8 1 7 9 5 1 8 5 7 7 1 0 5 0 9

we can transcribe these into staff music, say in the key of C, by a very simple numbering of the lines. An example is given below.



Any amateur pianist or guitarist can pluck out this melody with or without accompanying chords. To call this computer output *music* would be an exaggeration. One could just as easily use the digits of "pi" or of "*e*" or of "square root of 2" to produce a similar sequence. Indeed, this has been done.

Variation in duration, sharps, flats, etc. may also be introduced. We need to turn to a musicologist to determine the rules of acceptable music. Let us assume that we wish our computer to simulate the music of Bach. If a musicologist who studies the music of Bach can give us a set of rules which describe the structure of Bach's music, we may be able to program these rules into the computer and then test each random note against the rules which the musicologist provided. If a note satisfies the rules, we will accept it; if not, another note is generated and tested. Thus we can produce several pages of numerical jibberish which could be translated into line and staff music, which might or might not sound like the music of Bach. If it sounds like the music of Bach to several experts, we conclude that the rules of composition which our musicologist furnished described the structure of Bach's music. On the other hand, if the composition does not sound like Bach's music, then there is something missing from the set of rules which was supposed to describe Bach's music. In an actual experiment of this type, the musicologist was quite infuriated because the computer had placed the notes widely scattered over the 88 keys of the piano, whereas Bach composed for the clavichord which had a shorter keyboard. One of the recognizable features of Bach's music is that it uses the center portion of the piano keyboard. When the programmer asked the musicologist where his set of rules stipulated this, the musicologist shrugged it off and said, "Any moron knows that." Unfortunately, our computer did not know this. If you expect a computer to do a job, you must instruct it precisely, carefully, and accurately. The student who is interested in more work on computer music should first notice some of the work L. A. Hiller, Jr. and his group at the University of Illinois and the more recent work by Gerald Lefkoff describing a conference on computer applications in music held at West Virginia University in Morgantown, W.Va., in April, 1966. No doubt more recent work will be available within the next few years. A 1963 paper, *Xenakis* (by Iannis), which deals in greater length with the stochastic techniques, appeared in the 1963 issue of the journal *La Revue Musicale* under the title of "Musiques Formelles" published in Paris. A 1971 addition is Nonesuch H-71245 *Computer Music* by J. K. Randall, which contains several recordings. Columbia Masterworks recordings of *Switched on Bach* and *The Well-Tempered Synthesizer* by Walter Carlos also merit listening. Your library may have copies available.

The most important contribution which a study of this type has to offer is not the production of additional music, but rather that of assisting the musicologist in his task of analyzing the structure of Bach's music, or that of any other composer.

an excerpt from *Computer Programming: Techniques, Analysis and Mathematics* a text by the Andrees, published by Prentice Hall (1973)

Dr. and Mrs. Richard V. Andree, founders of Mu Alpha Theta, and son David with a copy of the new text the three of them coauthored.

THE TESSERACT (continued)

It may bother you that you still don't see four perpendiculars in either view of the tesseract; i.e., either Figure 4b or 5b. This really isn't strange at all. You don't really see three perpendiculars in Figure 2 or in Figure 6; you see the projection of the three perpendiculars. That is what we see in Figure 4 and 5; the projections of four perpendiculars.

There is another way to visualize the tesseract which might help. The method is analogous to thinking of a cube as being traced out by moving a square in a direction perpendicular to the sides of the square; that is, move the two-dimensional square in a third dimension.

Now we will try to move a cube. First start with the cube in Figure

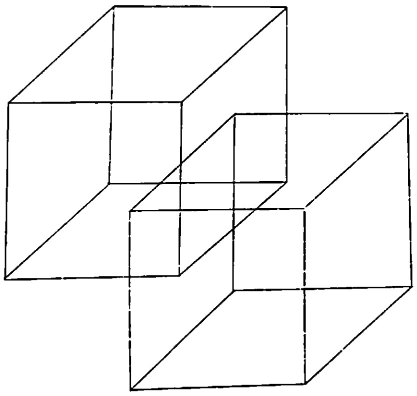


Figure 7a

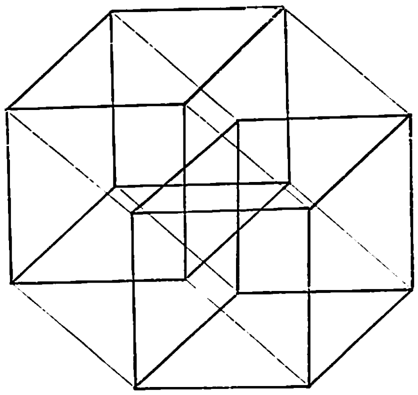


Figure 7b

7a. Think of it moving out from the paper and a little to the right. Actually we should move it in a fourth dimension, but this is hard to see. Figure 7b gives the final tesseract.

There are altogether eight cubes; the original cube in its initial and final positions, and the six cubes traced out by each of the six faces. These cubes are called *the bounding cells*. There are twenty four squares, each of which belongs to two cubes. There are thirty-two lines and sixteen vertices.

The bounding cells of a square are the four straight lines, the bounding cells of a cube are the six square faces [illegible] the bounding cells of the tesseract are the eight cubes. If we considered the five dimensional cube, it would have ten tesseracts as bounding cells.

There are, of course, many other four-dimensional objects. The four dimensional tetrahedron and octahedron are good examples and not too hard to draw if the reader wants to try his hand.

The concepts of fourth, fifth and even higher dimensions can be given very abstract treatments with concrete applications to physics and the real world. Other interpretations of the fourth dimension are much more down to earth and consider objects like gasses to be four dimensional when there are four variables in question. For example, in a gas the variables might be temperature, pressure, density and entropy. However, these interpretations are beyond the purpose of this article and are left for the interested reader to pursue.

Dr. David Ballew, S.D. School of Mines

SUMMER 1974

(Don't forget to plan NOW to attend The National Mu Alpha Theta Meeting in Fayetteville, Arkansas, August 4-7, 1974.)

PROBLEM SOLVING for fun and profit (continued)

that when N is a positive integer, then P(N) is a prime; a well-known quadratic polynomial

$$P(N)=N^2-N+41$$

takes on prime values for N=1, 2, 3, 4, ..., 39, 40, but clearly P(41) is not prime since it is >41 and divisible by 41. Thus, P(N) does not produce *only* primes nor does it produce *all primes*. I don't know whether it produces infinitely many primes or only a finite number of them. I do know that P(N)=4N—1 produces an infinite number of primes as well as composite values. Fermat (1601-65) thought $2^{2^N}+1$ produced only primes, but Euler (1707-83) showed that $2^{2^5}+1=4\ 294\ 967\ 297$ has a factor of 641. One must be rather careful how he poses the problem—if, for example, one wishes a polynomial formula P(N) which would produce a prime for each value of N it would be very easy to satisfy the stated desire (sure, you can do it—think a bit.).

On the other hand, if one insists that he really wishes a *non*-constant polynomial (degree one or greater), then it has been proved that any integral polynomial of degree one or greater cannot represent primes alone.

Indeed in Volume 53 (1946) of *American Math Monthly*, R. C. Buck showed that no rational function (quotient of two polynomials) of X other than a constant function can represent only primes for all positive intergral N.

Perhaps we should change our aim. Do you feel it would be difficult to find an integral polynomial which took on *every* prime value? (It would take on composite values, too, of course, but for each prime π_n there must be some positive integer N such that $P(N)=\pi_n$).

Problem 2

Create a formula such that when a positive integer N is substituted into the formula it will produce the N-th prime, or produce a proof that no such formula can exist.

Clearly, from what is given above, the formula cannot be a polynomial, nor even a rational function. I'll give you a helpful (*very* helpful) hint. There are such formulae—admittedly they are messy, but the fact that you know (if you believe me) that such formulae exist can be a big help to you. The one I'm thinking of involves several unusual properties, but the only thing it uses that you have not had is a Number Theoretic Theorem called Wilson's Theorem.

Well, get out your thinking hat. If you can't solve problem 2, see if you can at least find a function F(N) that will equal one if N is prime and zero if N is not prime. [Changing the problem is an old trick in the art of problem solving, and sometimes it helps solve the problem you wish solved to create a solution to a different but related problem—don't give up. This one didn't solve itself in a weekend.]

Problem 3

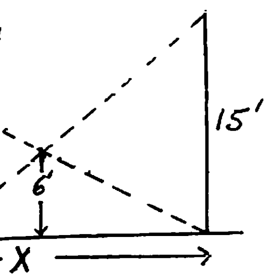
Page 26 of the 1969 *MAA Studies in Mathematics* Vol. 6, *Studies in Number Theory* book contains the statement, "The equation $x^3-117y^3=5$ is known to have at most 18 integral solutions, but the exact number of solutions is unknown." That is a fascinating statement and surely cries for further investigation—perhaps using a computer.

A program to try a few thousand values of X and Y can easily be written and tested. A more extensive program to try *all* integers $-10,000\leqq X, Y\leqq 10,000$ could also be written, but certainly *should not be used*, since many of the $4x10^8$ cases are *clearly* a waste of time—for example almost half of the cases have $x\geqq 2$ and $y\leqq 00$ in which case x^3-117y^3 is clearly greater than 5.

A bit of mathematical analysis seems worthwhile here. You can carry it out with the knowledge you have.

You now have three problems to investigate in any manner you deem appropriate. They are *quite different in approach*, and each illustrate a feasible technique of solution. We next examine three additional problems.

Problem 4



Two vertical poles are erected on level ground with a distance X between them. One pole is 10 feet tall, the other is 15 feet tall. The lines from the top of one pole to the base of the other pole cross at a point six feet above ground level. How far apart are the poles? (i.e., X=?)

Problem 5

Rearrange the letters in the phrase NEW DOOR to make one word. (In English, of course.)

Problem 6

It is well known that the length of the circumference of a circle can be approximated as accurately as desired by using the lengths of inscribed regular (equal sided) polygons. It is also well known that the circumference of a circle of radius r is $2\pi r$. These two facts may be combined to produce as close an approximation of π as is desired by approximating the length of the upper half of a circle of radius 1 using inscribed polygons. Write a computer program to approximate π in this fashion starting with the upper half of an inscribed hexagon (length of side=1, approximation of $\pi=3\cdot 1=3$) and doubling the number of sides in the approximating regular polygon.

Each of problems 4, 5, 6 represents a different type of problem solving than is present in the other problems. Try your hand at solving these six problems before the spring 1974 issue of *The Log* is delivered. Some are easy, others hard, but each represents a different problem solving technique.

THE MATHEMATICAL LOG

Volume XVII, No. 2 December, 1972

WHAT OPPOSITES TELL ABOUT A PERSON

To find the complement of a set S, we need to know the universe of discourse, or space U, under consideration. For example, the complement of the set

$$S = \{\text{coins of value } 25¢ \text{ and under}\}$$

is $\bar{s} = \phi$, the "null" (or no-membered) set, with respect to the space U of coins in my pocket right now, but the complement is

$$\bar{s} = \{50¢ \text{ pieces and silver dollars}\}$$

with respect to coins minted over the years.

Ordinarily we think of the space U and a set S as given, with the complement $\bar{s}$ unknown. What happens if we have a different unknown? What if we are given the set S and its complement $\bar{s}$, and it is the space U that is unknown? Theoretically, we can reconstruct the whole space U from just one set-complement pair, for $\bar{s}$ is made up of precisely those elements of U that are not members of S; then, $U = S \cup \bar{s}$, the union of all elements of S taken together with all elements of $\bar{s}$.

It is possible to guess something about the boundaries of an individual's little world (his "universe") by noticing what concepts he considers "opposites," that is, "complements." If your girl friend thinks the opposite of "animal" is "gentleman," that says something about her relationships with men. We might learn about an individual's experiences and impressions by posing a quiz on opposites. The most promising words for such a quiz are loose generic terms such as "good," "young," "long (hair, for instance)". We found that the word "green" has many different complements in different contexts or spaces: If someone answers "red" when asked for an opposite to "green", he may be thinking of strawberries or raspberries. If he says "blue", he may be thinking of blueberries. The complement of "green" is black for blackberries, orange for oranges, purple or white for lilacs. If the space U is lumber, then the complement of "green" is "seasoned". If someone offered "experienced" as an opposite for "green", we might guess that he had worked in an employment office, or some job that made him take "employees" as the universe with respect to which "green" required an opposite.

In the following list suppose the first word is given and experimental subjects offer the other word(s) in kind of a word anti-association quiz. See whether you can guess something about the person by finding his universe in which the words are opposites:

lightly	generously
easy	sudden
red	violet
billy	nanny
jack	jenny
vicious	finite
raw	reduced; cooked
conservative	liberal; radical
broken	solid; mended
verse	chorus; prose
true	magnetic; infidelitous; false; fictional
counting	delay; guessing
calm	windy; excited
lower	capital; human
propeller	jet; brake
smart	dowdy; well-behaved; stupid
egg	bird; bacon
typed	handwritten; dictated
composed	played; criticized; frantic
death	birth; life
strong	mild; weak
leave	keep; stay

We invite contributions to our list, especially words that have several different complements with respect to different universes.

Sallie Davis, Margaret Flanagan, Eddie Hsu, Mark Hulbert, Steve Hulbert, Tim Larsen, David Maxfield, John Nordin, Joe Poole, Nancy Reber, David Weixelman,
Manhatten Math Club, Manhatten, Kansas

COMPUTING IRRATIONAL SQUARE ROOTS

The problem of how to approximate $\sqrt{2}$ and other irrational numbers has been around at least as long as the Pythagorean Theorem. One of the easier ways to approximate $\sqrt{2}$ uses the theorem and a geometric construction to locate the corresponding point on the real number line.

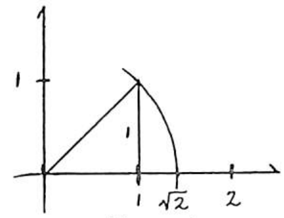


Figure 1

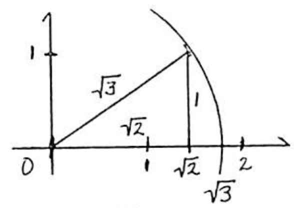


Figure 2

To locate $\sqrt{2}$, draw a cartesian coordinate system and construct the right triangle shown in figure 1. Then draw a circle with the center at the origin and radius equal to the length of the hypotenuse of the triangle. Since $1^2 + 1^2 = (\sqrt{2})^2$ this length approximates $\sqrt{2}$. A construction for $\sqrt{3}$ is shown in figure 2. Can you devise one for $\sqrt{5}$?

Notice that even the most careful work cannot obtain an approximation which is more accurate than the tools used. Techniques which use the algebraic properties of the real numbers are more likely to yield the desired number of digits of accuracy.

One scheme for finding an approximation for $\sqrt{2}$ is to choose a fraction N/D. Then, if $(N/D)^2 > 2$, increase the denominator; if $(N/D)^2 < 2$, increase the numerator. (Why is $(N/D)^2 \neq 2$? See *What is Mathematics*, Courant and Robbins). For example, if $N = 1$ and $D = 1$, then $N/D = 1$. Since $(N/D)^2 = 1$ and $1^2 < 2$, increase N by 1 so that the next approximation is $(1 + 1)/1 = 2$. Since $2^2 > 2$, increase the denominator by 1 so that $N/D = 2/2 = 1$. This procedure can be written as a BASIC program:

```
10 LET N = 1
20 LET D = 1
30 PRINT N/D;
40 IF (N/D)*(N/D) >= 2 THEN 70
50 Let N = N + 1
60 GOTO 30
70 D = D + 1
80 GOTO 30
:
RUN
1  2  1  1.5  1  1.3333333  1.6666667  1.25  1.5  1.2  1.4  1.6
1.3333333  1.5  1.2857143  1.4285714  1.25  1.375  1.5  1.3333333
1.4444444  1.3  1.4  1.5  1.3636364  1.4545455  1.3333333  1.4166667
:
```

The program was allowed to make 27 iterations, then halted manually. You can do this by hand if you remember that "N = N + 1" means "replace the value of N by the value of N + 1". If you have a computer available, be sure your program is more efficient than the one shown.

Techniques like this are called iterative processes. One iteration has been made each time N/D is changed. When the computer printed 1.5 in the output shown, it had completed three iterations. A speedier version of the program given obtained the approximation $N/D = 1.4142136$ in 5403 iterations. This is a *very* slow way to get $\sqrt{2}$ — it took our Wang 3300B BASIC nearly four minutes to get the result.

Another way to approach the problem is to choose a number $X_0^2 < 2$. Obviously $X_0 < \sqrt{2}$. To show that $\sqrt{2} < 2/X_0$,

Let $\quad X_0 < 2$

Then $\quad 1 < 2/X_0^2$, since $X_0 > 0$.

But $\quad 1 < \sqrt{2}/X_0$. Multiply both sides by $\sqrt{2}$,

$\quad \sqrt{2} < \sqrt{2}\sqrt{2}/X_0$;

Therefore $\quad \sqrt{2} < 2/X_0$.

Let $X_1 = (1/2)(X_0 + 2/X_0)$. Notice that X_1 is either closer to $\sqrt{2}$ than $2/X_0$, or closer to $\sqrt{2}$ than X_0. (Try a few values of X_0, and observe the values of $2/X_0$.) Define a recursion relation:

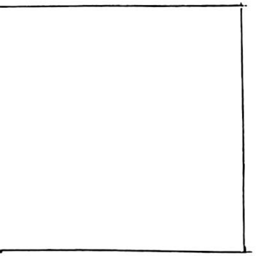
$$X_{n+1} = (1/2)(X_n + 2/X_n).$$

(See Mathematical Log article on Fibonacci numbers for more on re-

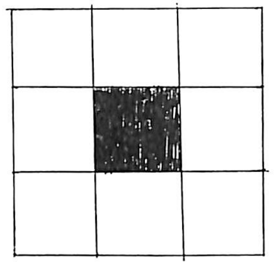
Continued on page 2

MATHEMATICAL DOODLES

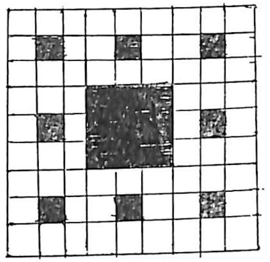
...After reading an article about mathematical doodles and Induction ...n the winter, 1971–1972 edition of Mu Alpha Theta's *The Mathematical ...og*, our Math Analysis class decided to do some doodling of our own. ...Ve devised what we named the Square Deal. Our first step was to draw ...he first five squares.



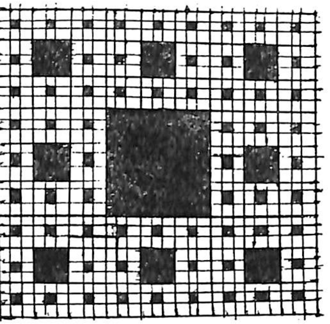
Square #1



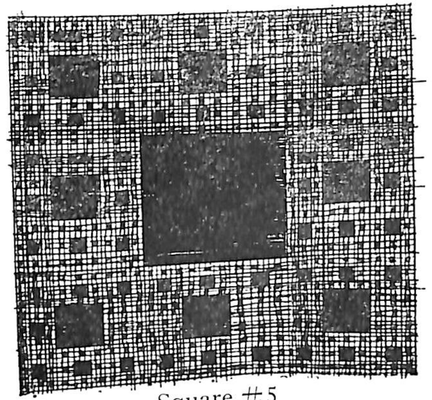
Square #2



Square #3



Square #4



Square #5

Our next step was to determine what portion of the squares was ...aded. We got 0, 1/9, 17/81, 217/729, and 2465/6561. Our curiosity ...

$= 8\,[8^2 + 8(9) + 9^2] + 729$ or $2465 = 8^3 + 8^2(9) + 8(9^2) + 9^3$. Substituting this information into our table, we had:

SQUARE #	NUMERATOR
1	0
2	1
3	$8^1 + 9^1$
4	$8^2 + 8(9) + 9^2$
5	$8^3 + 8^2(9) + 8(9^2) + 9^3$
. . .	. . .
k	?

Comparing the square number and the numerator, we noticed that each numerator is the sum of the products of 8 and 9 raised to different powers. The first summand is 8 raised to a power of 2 less than the square number times 9 to the 0. In the succeeding summand the power of 8 is diminished by 1 while the power of 9 is raised by 1. Therefore, the numerator of Square k will be $8^{k-2} + 8^{k-3}(9) + \ldots + 8(9^{k-3}) + 9^{k-2}$. Turning to our knowledge of factoring binomials, we remembered that $(8^2 - 9^2)/(8 - 9) = 8 + 9$ or $(8^2 - 9^2)/(-1) = 8 + 9$; thus $9^2 - 8^2 = 8 + 9$. Using the same method $(8^3 - 9^3)/(8 - 9) = 8^2 + 8(9) + 9^2$ or $9^3 - 8^3 = 8^2 + 8(9) + 9^2$; therefore $9^{k-1} - 8^{k-1} = 8^{k-2} + 8^{k-3}(9) + \ldots 8(9^{k-3}) + 9^{k-2}$. Amending our table, we had:

SQUARE #	NUMERATOR
1	0
2	1
3	$9^2 - 8^2$
4	$9^3 - 8^3$
5	$9^4 - 8^4$
. . .	. . .
k	$9^{k-1} - 8^{k-1}$

Therefore, the numerator of Square k will be $9^{k-1} - 8^{k-1}$. Combining this with the denominator we obtained, we had the portion of Square k that was shaded in. It equaled: $(9^{k-1} - 8^{k-1})/(9^{k-1})$ or $(9^{k-1})/(9^{k-1}) - (8^{k-1})/(9^{k-1})$ or $1 - (8/9)^{k-1}$.

Unwilling to leave well enough alone, we devised what we called the Dirty Deal. We decided to try to find a formula that would work for any polygon that can be divided into n equal parts. First, we drew a

POLYGON	DENOMINATOR
1	?
2	n
3	n^2
4	n^3
5	n^4
. . .	. . .
k	?

Combining what we learned from our work on the Square Deal with a close scrutiny of our table, we came to the conclusion that each denominator for Polygon k would be n^{k-1}.

Next we turned to the difficult task of finding the numerator for Polygon k. Again we drew a table.

POLYGON #	NUMERATOR
1	0
2	1
3	$(n-1) + n$
4	$(n-1)^2 + n(n-1) + n^2$
5	$(n-1)^3 + n(n-1)^2 + n^2(n-1) + n^3$
. . .	. . .
k	?

Comparing this table with the one we used to find the numerator of the Square Deal, we found certain similarities. Each table involves the sum of a number n and n–1 raised to a certain power. We also noticed that each term of the resulting binomial was a product of n and n–1. Furthermore, each binomial starts with $n^{k-2}(n-1)^0$ and progresses with the power of n decreasing each term while the power of n–1 increases until the last term equals $n^0(n-1)^{k-2}$. From these observations we could see that the numerator of Polygon k will be $n^{k-2} + n^{k-3}(n-1) + \ldots + n(n-1)^{k-3} + (n-1)^{k-2}$.

Leaning again on our knowledge of factoring binomials, see $((n-1)^2 - n^2)/((n-1) - n) = (n-1) + n$, and $((n-1)^3 - n^3)/((n-1) - n) = (n-1)^2 + n(n-1) + n^2$. Thus, $((n-1)^{k-1} - n^{k-1})/((n-1) - n) = (n-1)^{k-2} + (n-1)^{k-3}\,n + \ldots + n^{k-3}(n-1) + n^{k-2}$. Next we changed our table to read:

POLYGON #	NUMERATOR

THE MATHEMATICAL LOG

December, 1972 Volume XVII, No. 2

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ANNUAL MATHEMATICS CONTEST IS SCHEDULED FOR MARCH 13

The Annual High School Mathematics Contest, sponsored by Mu Alpha Theta, the National Council of Teachers of Mathematics, the Mathematical Association of America, the Society of Actuaries, and the Casualty Actuarial Society is to be held on March 13, 1973. It is again included on the official activities list of the National Association of Secondary School Principals (U.S.A.).

Registration, which is handled regionally, closes January 15. Write now to Henry M. Cox, 1222 Seaton Hall, 15th and U Streets, Lincoln Nebraska 68508, for regional applications and order forms for recent examinations and solution-keys.

Questions and solutions for the years 1961–65 are contained in The M.A.A Contest Book II, volume 17 of the New Mathematics Library, published by Random House/L.W. Singer Co. for the School Mathematics Study Group.

COMPUTING IRRATIONAL SQUARE ROOTS Cont.

cursion relations.) If $X_0 = 1$, then $X_1 = (1/2)(1 + 2/1) = 1.5$ and $X_2 = (1/2(1.5) = 1.42$. This can be written in BASIC as

```
10 LET X = 1
20 LET X = (1/2)*(X + 2/X)
30 PRINT X;
40 GOTO 20
:
 RUN
 1.5   1.4166667   1.4142157   1.4142136   1.4142136   1.4142136
3300 BASIC READY
:
```

This algorithm is much faster—it approximates $\sqrt{2}$ to eight decimal places in only four iterations. (Again, the program was halted manually.)

Some questions which may help in further investigation are:

1. What other starting values for N, D, and X_0 could be chosen to speed "convergence"?
2. How can these techniques be modified to approximate other irrational square roots? What does $X_{n+1} = (1/2)(X_n + 3/X_n)$ do?
3. How should these techniques be modified to approximate cube roots or fourth roots?

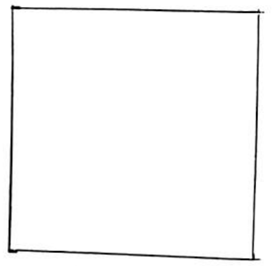
Finally, to find out more about the second technique, look up Newton's Method or the Newton-Raphson Method in an elementary calculus book.

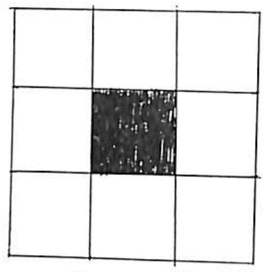
An interesting, and ancient, technique is that of "side-and-diagonal numbers", for which see Maxfield, J. E. and M. W., *Discovering Number Theory*, W.B. Saunders Co., Inc., 1972, pp. 92-94, or F. V. Waugh & M. W. Maxfield's "Side-and-Diagonal Numbers", *Math Magazine*, Vol. 40, No. 2, March 1967, pp. 74-83.

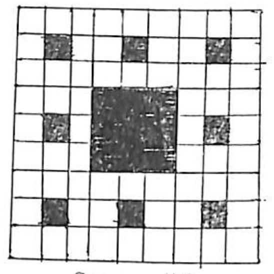
Paula Vitasek May
Oklahoma University

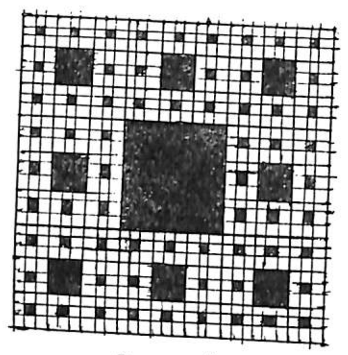
MATHEMATICAL DOODLES

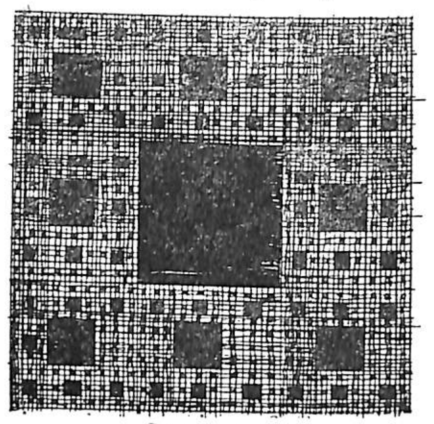
After reading an article about mathematical doodles and Induction in the winter, 1971–1972 edition of Mu Alpha Theta's *The Mathematical Log*, our Math Analysis class decided to do some doodling of our own. We devised what we named the Square Deal. Our first step was to draw the first five squares.


Square #1


Square #2


Square #3


Square #4


Square #5

Our next step was to determine what portion of the squares was shaded. We got 0, 1/9, 17/81, 217/729, and 2465/6561. Our curiosity aroused, we decided to try to find what portion of Square k would be shaded in. To do this, we had to find a relationship between each shaded fraction. Since the denominators looked easier to figure out than the numerators, we started with them. To get a better look at our information, we drew the following table:

SQUARE #	DENOMINATOR
1	?
2	9
3	81
4	729
5	6561
. . .	. . .
k	?

Looking at this table, we asked, "How are 9, 81, 729, and 6561 related?" Each number is odd and divisible by 9. Furthermore, they are all powers of 9. Thus we amended our table to read:

SQUARE #	DENOMINATOR
1	$1 = 9^0$
2	$9 = 9^1$
3	$81 = 9^2$
4	$729 = 9^3$
5	$6561 = 9^4$
. . .	. . .
k	?

From this table we saw that each denominator was 9 raised to the power of one less than the square number. Therefore, it is easy to see that the denominator of Square k will be 9^{k-1}.

Next, we embarked on our search for the numerator of Square k. Again we drew a table.

SQUARE #	NUMERATOR
1	0
2	1
3	17
4	217
5	2465
. . .	. . .
k	?

Again we were confronted with the problem of finding a relationship among these numbers. Through trial and error we found that $217 = 8(17) + 81$ and $2465 = 8(217) + 729$. Since $17 = 8 + 9$, we can also say that $217 = 8(8 + 9) + 81$ or $217 = 8^2 + 8(9) + 9^2$. Thus $2465 = 8[8^2 + 8(9) + 9^2] + 729$ or $2465 = 8^3 + 8^2(9) + 8(9^2) + 9^3$.

Substituting this information into our table, we had:

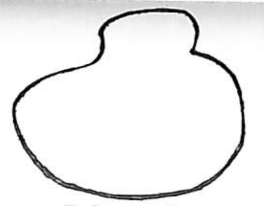
SQUARE #	NUMERATOR
1	0
2	1
3	$8^1 + 9^1$
4	$8^2 + 8(9) + 9^2$
5	$8^3 + 8^2(9) + 8(9^2) + 9^3$
. . .	. . .
k	?

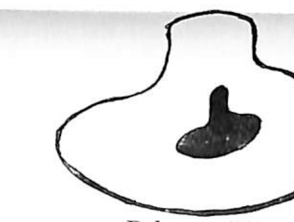
Comparing the square number and the numerator, we noticed that each numerator is the sum of the products of 8 and 9 raised to different powers. The first summand is 8 raised to a power of 2 less than the square number times 9 to the 0. In the succeeding summand the power of 8 is diminished by 1 while the power of 9 is raised by 1. Therefore, the numerator of Square k will be $8^{k-2} + 8^{k-3}(9) + \ldots + 8(9^{k-3}) + 9^{k-2}$. Turning to our knowledge of factoring binomials, we remembered that $(8^2 - 9^2)/(8 - 9) = 8 + 9$ or $(8^2 - 9^2)/(-1) = 8 + 9$; thus $9^2 - 8^2 = 8 + 9$. Using the same method $(8^3 - 9^3)/(8 - 9) = 8^2 + 8(9) + 9^2$ or $9^3 - 8^3 = 8^2 + 8(9) + 9^2$; therefore $9^{k-1} - 8^{k-1} = 8^{k-2} + 8^{k-3}(9) + \ldots 8(9^{k-3}) + 9^{k-2}$. Amending our table, we had:

SQUARE #	NUMERATOR
1	0
2	1
3	$9^2 - 8^2$
4	$9^3 - 8^3$
5	$9^4 - 8^4$
. . .	. . .
k	$9^{k-1} - 8^{k-1}$

Therefore, the numerator of Square k will be $9^{k-1} - 8^{k-1}$. Combining this with the denominator we obtained, we had the portion of Square k that was shaded in. It equaled: $(9^{k-1} - 8^{k-1})/(9^{k-1})$ or $(9^{k-1})/(9^{k-1}) - (8^{k-1})/(9^{k-1})$ or $1 - (8/9)^{k-1}$.

Unwilling to leave well enough alone, we devised what we called the Dirty Deal. We decided to try to find a formula that would work for any polygon that can be divided into n equal parts. First, we drew a polygon.


Polygon #1


Polygon #2

We then drew another, similar polygon, dividing it into n equal parts, and shading in one of these.

We then drew three more similar polygons, dividing each into n equal parts, shading one of these and then equally dividing each remaining part.


Polygon #3


Polygon #4


Polygon #5

Next we computed what portion of the polygons was shaded in. The first two were easy. They were 0 and 1/n, respectively. The next three posed a problem. In Polygon 3 there were n–1 equal parts remaining. We divided each n part n times, making each portion $1/n^2$ in size. Therefore, Polygon 3 had $(1)/(n) + (n-1)/(n^2)$ parts shaded in; or $((n-1) + n)/(n^2)$. In Polygon 4, we divided the n–1 remaining n^2 parts n times making each portion n^3 in size. Adding these $((n-1)^2)/(n^3)$ portions to the shaded portion in Polygon 3, we found that in Polygon 4 $(n[(n-1) + n] + (n-1^2)/(n^3)$ would be shaded. Using the same procedure on Polygon 5, we found that the shaded portion would equal $(n[n(n-1 + n) + (n-1^2] + (n-1)^3)/(n^4)$. Since we knew what the shaded portions of the first five polygons would be, we decided to find what portion of Polygon k would be shaded. We decided to find the denominator first, since it appeared to be easier. Our first step was to draw the following table:

POLYGON	DENOMINATOR
1	?
2	n
3	n^2
4	n^3
5	n^4
. . .	. . .
k	?

Combining what we learned from our work on the Square Deal with a close scrutiny of our table, we came to the conclusion that each denominator for Polygon k would be n^{k-1}.

Next we turned to the difficult task of finding the numerator for Polygon k. Again we drew a table.

POLYGON #	NUMERATOR
1	0
2	1
3	$(n-1) + n$
4	$(n-1)^2 + n(n-1) + n^2$
5	$(n-1)^3 + n(n-1)^2 + n^2(n-1) + n^3$
. . .	. . .
k	?

Comparing this table with the one we used to find the numerator of the Square Deal, we found certain similarities. Each table involves the sum of a number n and n–1 raised to a certain power. We also noticed that each term of the resulting binomial was a product of n and n–1. Furthermore, each binomial starts with $n^{k-2}(n-1)^0$ and progresses with the power of n decreasing each term while the power of n–1 increases until the last term equals $n^0(n-1)^{k-2}$. From these observations we could see that the numerator of Polygon k will be $n^{k-2} + n^{k-3}(n-1) + \ldots + n(n-1)^{k-3} + (n-1)^{k-2}$.

Leaning again on our knowledge of factoring binomials, see $((n-1)^2 - n^2)/((n-1) - n) = (n-1) + n$, and $((n-1)^3 - n^3)/((n-1) - n) = (n-1)^2 + n(n-1) + n^2$. Thus, $((n-1)^{k-1} - n^{k-1})/((n-1) - n) = (n-1)^{k-2} + (n-1)^{k-3}n + \ldots + n^{k-3}(n-1) + n^{k-2}$. Next we changed our table to read:

POLYGON #	NUMERATOR
1	0
2	1
3	$\frac{(n-1)^2 - n^2}{(n-1) - n}$
4	$\frac{(n-1)^3 - n^3}{(n-1) - n}$
5	$\frac{(n-1)^4 - n^4}{(n-1) - n}$
. . .	. . .
k	$\frac{(n-1)^{k-1} - n^{k-1}}{(n-1) - n}$

Combining the results that we obtained for the numerator and denominator, we found that the shaded portion of any Polygon k that can be divided in n equal parts will be:

$$\frac{\frac{(n-1)^{k-1} - n^{k-1}}{(n-1) - n}}{n^{k-1}} \quad \text{or} \quad \frac{n^{k-1} - (n-1)^{k-1}}{n^{k-1}} \quad \text{or}$$

$$\frac{n^{k-1}}{n^{k-1}} - \frac{(n-1)^{k-1}}{n^{k-1}} \quad \text{or} \quad 1 - \frac{(n-1)}{n}^{k-1}$$

Linda McDonald, student
Ada High School, Ada, Oklahoma

U.S.A. MATH OLYMPIAD

The First U.S.A. Mathematical Olympiad was given to selected high school students on May 9, 1972. We thought you would enjoy looking at the problems and trying a few. No one was expected to do all of the problems in the three hour period, but contestants were expected to do a good job on those submitted.

1. The symbols (a,b, . . . ,g) and [a,b, . . . ,g] denote the greatest common divisor and the least common multiple, respectively, of the positive integers a,b, . . .g. For example, (3,6,18) = 3 and [6,15] = 30. Prove that

$$\frac{(a,b,c)^2}{(a,b)(b,c)(c,a)} = \frac{[a,b,c]^2}{[a,b][b,c][c,a]}$$

2. A given tetrahedron ABCD is isosceles, that is, AB = CD, AC = BD, AD = BC. Show that the faces of the tetrahedron are acute-angled triangles.

Continued on page 4

Trial							
1	12	3	10	43	51	20	No
2	13	38	36	3	21	39	No
3	47	49	15	35	19	42	No
4	41	21	31	39	46	35	No
5	39	30	2	36	45	18	No
6	18	18	50	7	31	32	Yes
7	44	9	41	18	18	12	Yes
8	49	42	52	2	50	7	No
9	25	6	5	35	45	10	No
10	43	43	6	36	24	49	Yes
11	1	9	38	8	19	10	No
12	15	31	46	13	2	23	No
13	22	7	32	38	17	28	No
14	28	32	39	38	26	46	No
15	29	48	11	12	38	48	Yes
16	19	29	3	11	49	4	No
17	5	42	47	20	44	52	No
18	24	7	20	28	5	17	No
19	29	3	31	29	18	18	Yes
20	37	32	1	43	39	7	No
21	14	14	27	38	8	50	Yes
22	37	21	14	42	24	33	No
23	[illegible]	30	6	34	16	46	No
24	12	41	43	[illegible]	9	17	[illegible]

Table I

6	11	9	12	6	8	1	No
7	9	7	11	5	2	3	No
8	5	7	1	11	12	11	Yes
9	12	4	11	8	9	7	No
10	4	5	11	12	5	1	Yes
11	4	1	12	4	5	3	Yes
12	5	7	7	11	13	8	Yes
13	11	3	8	9	6	8	Yes
14	11	5	11	12	11	2	Yes
15	8	6	3	4	12	2	No
16	4	11	6	1	4	12	Yes
17	6	10	1	1	10	8	Yes
18	2	12	5	12	10	5	Yes
19	1	5	7	9	6	12	No
20	1	10	9	1	4	8	Yes
21	10	4	5	9	7	1	No
22	5	9	5	8	4	7	Yes
23	3	11	13	1	7	13	Yes
24	13	11	3	6	13	13	Yes

Table II

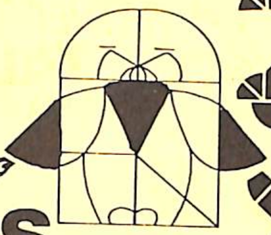
This problem can also be modified by using a partial deck. E.g., if each of the 6 students has a deck which contains 3 suits or 39 cards, what is the probability of a match?

You can clearly make many other modifications of this problem. For an interesting example, see the article by Richard S. Kleeber, "A Classroom Illustration of a Nonintuitive Probability", in *The Mathematics Teacher*, May, 1969.

Elliot A. Tanis, Hope College, Holland, Michigan

TALL TIMBERS #4 ... CONVENTION, BACK-TO-SCHOOL ISSUE

MU ALPHA THETA'S CHAPTER-ORIENTED PUBLICATION.

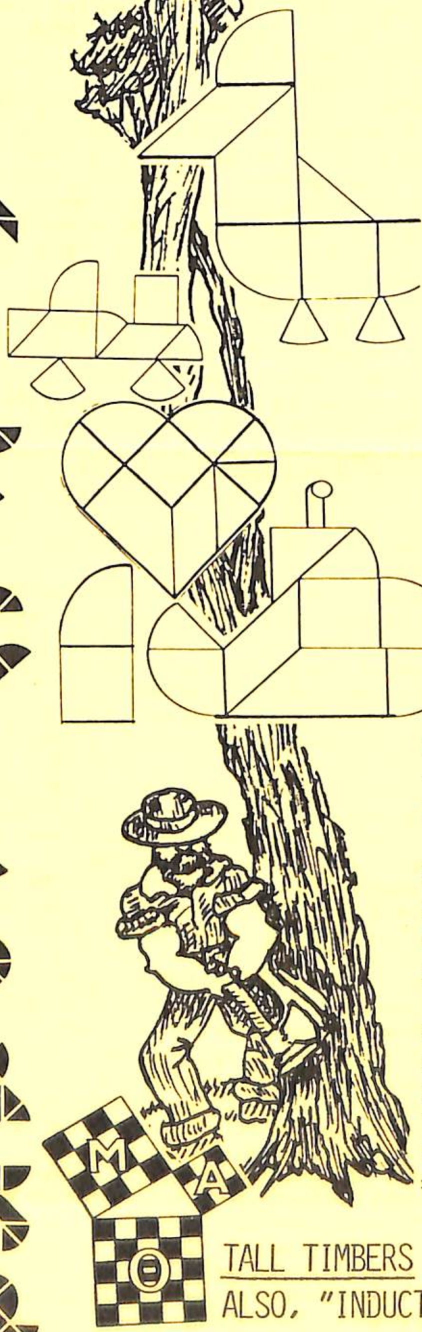


MATHEMATICAL TALL TIMBERS

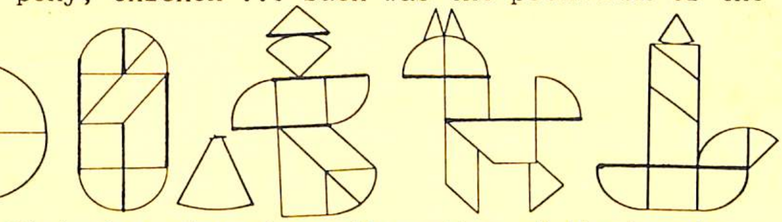
CHAPTER SUPPLEMENT TO THE MATHEMATICAL LOG

VOLUME 27, NUMBER 5 OCTOBER 1983

Dissection 'Great Fun' Chapter Reports



"Great fun!" reports the Mu Alpha Theta chapter at Baymonte Christian School, Scotts Valley, CA. "Our chapter had great fun with the Valentine dissection puzzle," writes Paul T. Blocher, Mathematics Chairman. Enclosed, as eloquent evidence, was an envelope of colorful, pasted-up original creations. Jill Clark's Very Important Penguin had a beak rising into a third dimension. Scott Walden created a Courier Service vehicle with quarter-circle "wheels." Jon Walden offered a modernistic submarine. Candle, pony, chicken ... such was the potential of the



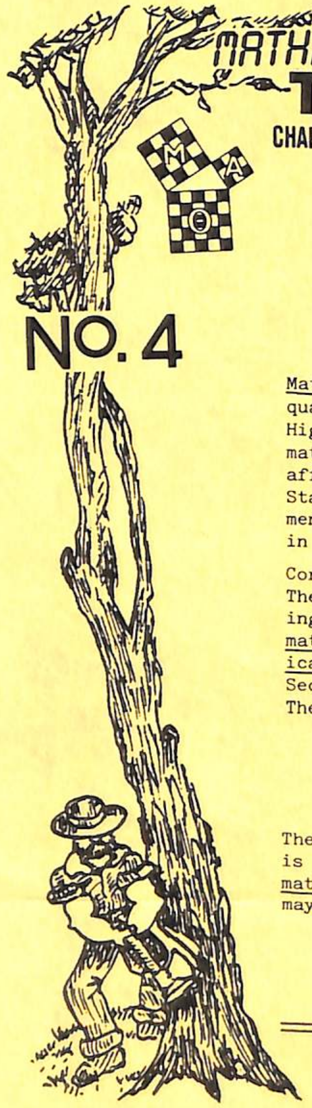
Victorian nine-piece dissection of the heart that was The Mathematical Log Valentine challenge (February Log, p. 4).

"Heartbreaker," as the dissection activity has been promoted and merchandised, was a clever nineteenth-century variant on the tangram dissection, and special fun for those for whom tangrams already were familiar.

Further challenge? The sky's the limit, one might say. In which spirit we've "bordered" our Tall Timbers front page with other heart dissection "possibles" ... but strongly suspect that the greatest fun is coming up with original figures of one's own.

TALL TIMBERS COLUMNIST DEBBIE PATONAI ON CHAPTERS ... PAGE 2.

ALSO, "INDUCTION" POETRY ... FOODLE DOODLE CURVE ... THIS ISSUE.



MATHEMATICAL TALL TIMBERS

CHAPTER SUPPLEMENT TO The Mathematical Log

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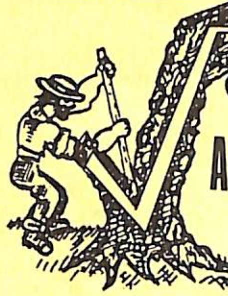
Editorial Greetings

Welcome back ... to another ever-busy, never-dull year of Mu Alpha Theta, and to Mathematical Tall Timbers, your Log supplement, linking 1200 active chapters in the U.S. and abroad. We've a packed issue--which always pleases us!--and urge you to keep news and views coming ... as, for instance:

Our "Prime Guess on a Postcard" Contest (April "diaLogue") has yielded, as expected, a record influx of Log-addressed postcards, but the most interesting card to date was not a contest entry (they're still arriving!) but a Christmas-vacation travel souvenir. From Carol Chenaud in Texas comes a picture and story of a visit to the world's first 3-D maze--in Wanaka, South Island, New Zealand. "It looks relatively simple, but the average maze-solver takes 30 minutes," Carol reports. She spent nearly two hours ... in part "playing with all the puzzles they sell there." Carol's other down-under math-related diversion: "counting sheep!"

* * *

Lee Veal, Walton High School, Marietta, GA; Joe Cates, Ft. Deposit, AL; and Steven Seelig, Friendswood, TX (a sophomore) were among those forwarding fairly extended solutions to Zal Usiskin's splendid "Three 3s" challenge (December Log, p. 6). The "last word" on that contest has yet to be heard, we now realize.



CHAPTER WISE

AT THE ROOT OF IT ALL

with Deborah Patonai

Reflecting and seeking to reinforce the unique "chapter" structure of Mu Alpha Theta, The National High School and Junior College Mathematics Club, this regular column feature of Mathematical Tall Timbers, chapter supplement to The Mathematical Log, welcomes and invites your input of news, opinions, and program ideas for school and college clubs. Assistant Editor Deborah Patonai looks forward to hearing from you. Write her ... at St. Vincent-St. Mary High School, 15 N. Maple St., Akron, OH 44303. --Ed.

Miss Patonai observes:

Mathematics contests and competitions are focal points of many Mu Alpha Theta chapter annual agendas, and the high point of an active and diverse "math club" year. Through such competition, many schools have earned local and state recognition for mathematical expertise. Planning begins in early Fall for the teams, many of which climax their year of hard work at National Convention.

Students at Kamehameha High School, Honolulu, HI, experience a unique opportunity to prepare for their mathematics competitions. Early each Fall, 25 students and 5 math teachers spend a weekend together preparing for upcoming contests. This "Math Camp," we understand, has been successful in helping the students know each other and work together, integrating (as it has) concentrated study sessions, competitions, and social activities. Just imagine ... studying quadratic equations for an hour and a half, with the special incentive a swim off some beautiful Hawaiian beach!

Traveling now to Orange, TX, Mu Alpha Theta's West Orange High School chapter was, we hear, very busy in January. The club sponsors a yearly Mu Alpha Theta math tournament for high schools and middle schools. West Orange hosted about 75 schools this time, distributed over 2000 tests, and gave out 91 trophies in various test divisions. Tests were given in Number Sense, Calculator Applications, Algebras I and II, Geometry, Advanced Math (high school), and Middle School Math, along with High School and Middle School Science. Sounds like an enormous--and very worthwhile--chapter and school undertaking!

In Milwaukee, WI, area, we hear, 12 Mu Alpha Theta chapters work together on their own mini convention. This past March, at Marquette University, over 250 students and advisorsparticipated in this year's gettogether. Included in all-day activities were an individual contest, student and adult presentations, and a "math bowl" won by Milwaukee Rufus King.

Special recognition, too, has been earned by several other schools which have topped recent competitions in their areas or states. Congratulations go to Riverdale High School, Ft. Myers, FL, on winning first place in Florida's Alpha Bowl competition. Congratulations, too, to Senior Scholarship winners at Texas State Mu Alpha Theta Convention: first place, Truman Joe of Klein High School (a Logmaster, the Editor tells me); second place, L. Quan Vu of Rider High School; and third place, Kerry Go of Bellaire High School. Similarly, in Arkansas, this column's congratulations go to Marion High School ... Marion came away with first place honors in the Hughes High School Invitational Math Contest, sponsored by Hughes High School chapter of Mu Alpha Theta.

Mathematics-Con and Pro

'Game of all Games' Lauded in Verse At Missouri Mu Alpha Theta Induction

Mu Alpha Theta chapters do some very good things at Inductions, we know--and we've always felt that such "good things" should be shared. Accordingly, Mathematical Tall Timbers is delighted to have received from Bonnie Coleman and Ron Rogers, Mu Alpha Theta sponsors at Parkway North High, Creve Coeur, MO, the full text of "Mathematics--Con and Pro," a 26-verse poem by Tim Callahan, "an outstanding member of Mu Alpha Theta ... for the past two years." Tim becomes our newest Logmaster on the strength of this effort, and the text of his original Mu Alpha Theta poem follows.

Math'matics and anarchy--one and the same
They both make a mess of the simplest of games.
Though one makes a system; the other one not,
The chaos they cause sure seems like a lot.

While hunting for epsilon's, delta's, and mu's
Probability states that you're going to lose
This game of all games: this study of numbers;
You find yourself drifting off slowly to slumber.

You battle with sleep as you try to awaken.
You look at your page--your equation's mistaken!
You'll have to rework it all over again!
"What's (a+b) quantity raised to the ten?"

You struggle; your forehead is covered with sweat.
Your hair stands on end, but you're not finished yet.
Your mind starts to wander; your nerves are all tense.
You look at the problem--it doesn't make sense!

You're filled with some awe, but mostly just fear.
"Find the volumes of cones intersected with spheres."
You see the next problem--it just isn't fair:
"Find a regular pentagon drawn in a square."

You wipe off your brow; your thinking is frantic.
You're fed up with doing Pacific-Atlantic.
Your fingers are shaking; you're filled with fatigue.
You don't care about the Missouri Math League.

Math'matics is sloppy--just look at the sty.
Why have an irrational number for pi?
It's ruined the numbers; the system's been debased.
And look at the natural logs--they're all e-based!

Look at Pythagoras--there is a man
Who tries to make trouble wherever he can.
His theorem's a bother; there's also no doubt
That squares may be in, but square roots are out!

Look at the guy who invented the zero:
He certainly shouldn't be thought as a hero.
Making that thing was not something he aught.
For all of his work, all he got was a naught.

Euclid--another Greek--one of the greats
Whose destiny seemed to be shaped by the Fates.
The timbers of many a problem he felled,
And as geometrician, he's unparalleled

Fermat, a lawyer, would go in his attic,
Sit down, alone, and do mathematics.
He hunted down mysteries, hiding aloof.
He had the answer, but where was the proof?

Gauss--you'll find him all over in lit'rature
Gaussian curvature, Gaussian quadrature
Gauss as in magnets and even inductance.
He should be studied with greatest reluctance.

Newton, we prob'ly can say, was the worst
'Tween Leibnitz and him, he made Calculus first.
That torture imposed upon students unwilling.
For all of his time he got only half billing.

Euler (or "yooler") for answers went fishin'
And quickly became a geometrician.
The Konigsberg problem, he did that one too
And found "v+f-e=2."

Kepler: he might be considered a jerk.
He watched the night sky, and he called that his "work."
Compared to the others he's not such a beast.
In fact it is said that of squares, he's the least.

Another great wizard of math was Descartes.
He tried to turn Algebra into an art.
His pictures were bad, but his papers had room
To scribble a quick "Cogito ergo sum."

Pascal was a man who was always quite sour.
He tried to raise (a+b) all to one power.
He also invented the Pascal Triangle.
And thus helped to make mathematics more mangled.

Mobius some might defend as quite grand
'Twas he who invented the Mobius band.
But those who defend this man seem to have missed it.
This was a person whose mind was half-twisted.

Is Mobius strange?--well, he's different, you bet!
But mathematicians get far stranger yet.
One of the strangest was called Felix Klien.
Whose bottle would freak out the calmest of minds.

It loops to itself and it bores its way through.
As though it had no idea what it should do.
No idea where it's going, and none where it's been
Perhaps inside-out, and perhaps outside-in.

Godel was not known for handwriting neatness.
He scrawled out his Theorem of Grand Incompleteness.
This was the worst that he ever could do--
His theorem disproves everything that is true.

Heisenberg, certainly, wins an award
For having less sense than a bucket of lard.
In science the role of precision is hurtin'.
It doesn't look good, but its future's Uncertain.

The prize goes to Einstein, who tried to bend space
And wound up by putting his name in disgrace.
Varying time as he gained further speed--
This kind of garbage we just do not need!

Math'matics, you see, is a whole lot of bunk,
And I don't see why we put up with the junk.
Long-hailed as a wonder, often called grand--
Math'matics--a system that ought to be banned.

And yet, there's a certain appeal to the stuff.
It's like an addiction--you can't get enough.
Math'matics, though sloppy, is like a disease:
It makes your head swim and it's likely to please.

By now you're bored out of your skull--I can tell
And yet I must leave you a final farewell.
As you go through your courses, attack them with zeal.
May your powers be even, your numbers all real.

FUN IN SIXES

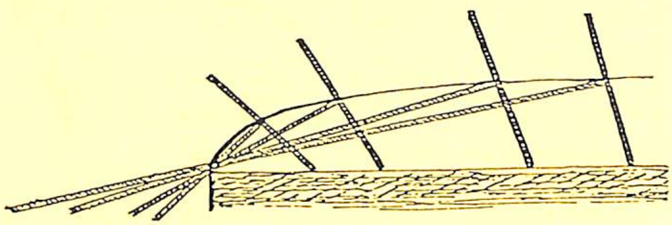
"What are you working on?" That's a question that those privileged to indulge in mathematics, as work or as, say, sport, do get asked. "In base six, find an integer whose 6-digit square contains the six digits," posed by Alan Wayne recently in School Science and Mathematics, was giving us fun at presstime. Such a number proved easy enough to find, but elegantly showing its uniqueness for us has been the real challenge. H.D.A.

Unbounded Region, Finite Area
Charm of 'Foodle Doodle' Curve

By Isaac Ji Kuo
Baton Rouge, LA

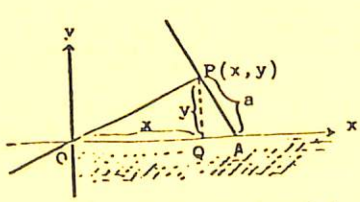
Have you ever heard of a curve called "Foodle Doodle"? Well, I bet you haven't, because I am the person who made up the name.

It all happened one Wednesday evening while my younger brother, Henry, and I were watching a TV show, "Fall Guy." I was also fooling (or rather "foodling," which is a word I made up for the name of the curve) with a T-square ruler on a table, and asked myself what kind of curve the joint point would make. The curve looks like the following:



Then I thought in my mind that at one end (the table edge) the curve looked like a parabola and at the other end (at "infinity") it looked like a hyperbola. I got so excited that I went to ask my Daddy right away to see what the curve was called. Well, he was quite busy with his work then, and told me that he would give me the answer later on.

The next morning, my Daddy showed me the equation of the curve, which he had derived. He told me that there was no special name for the curve, and said that I could give it a name, as I liked it. Actually, he got excited about the curve, too. Well, I just thought for a second, and the name "Foodle Doodle" popped out from my head. My Daddy told me that it was a very cute name. That evening, he explained to me how he obtained the equation of the curve, "Foodle Doodle." I understood it. Well, not quite. But, finally, I figured it out. The derivation is as follows:

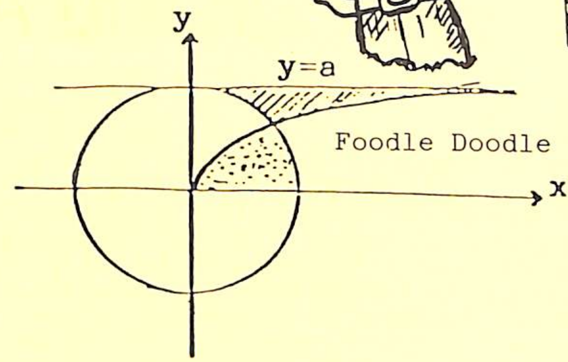


$\triangle OPQ \sim \triangle PQA$ (similar triangles)

$$\therefore \quad \frac{x}{y} = \frac{y}{\sqrt{a^2-y^2}}$$

$$\therefore \quad x = \frac{y^2}{\sqrt{a^2-y^2}}$$

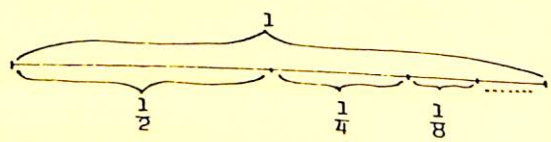
My Daddy told me something very interesting, namely, the area bounded by the "Foodle Doodle," the horizontal line $y = a$, and the y-axis is one-fourth of the area of a circle with radius a. Therefore, the shaded region and the dotted region have the



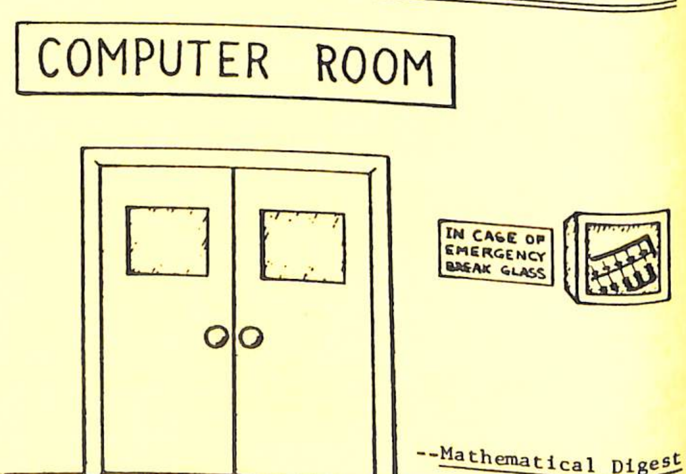
same area. This is very amazing because the shaded region is unbounded (!) and yet has finite area. Well, I was not totally surprised to learn this fact because I had already known for a long time that

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots = 1;$$

that is,



I asked my Daddy how he found out the area. He told me that he used Calculus. I asked him what Calculus was. He just said that he would explain it to me someday. Oh well, I guess I will have to wait. But, for the time being, I have my curve, "Foodle Doodle"!



--Mathematical Digest

NOT A LOGGER!

I never used a logarithm in my life, and could not undertake to extract the square root of four without misgivings.

--George Bernard Shaw.

Isaac Ji Kuo, our newest and youngest Logmaster, is a Grade V pupil at Buchanan School in Baton Rouge. His Daddy is Professor of Mathematics at Louisiana State University. --Ed.

CONVENTION, BACK-TO-SCHOOL ISSUE

The Mathematical Log

VOLUME 27, NUMBER 5 OCTOBER 1983

'Elementary' Math Not Necessarily 'Easy' Victorian-Era Exam Problems Reveal

By Don Allen

(For Presentation at our Oklahoma Convention)

Creating, uncovering, solving, and sharing mathematical "problems" having just the right level of challenge and difficulty has to be among the most satisfying and personally rewarding of math-related individual and small-group activities. The "story problem" (something of a lost art form!), has special appeal in this connection, capturing as it can the mathematical essence of a more-or-less real-world situation, then offering it for interpretation and "model solution." "Lost art" or not, the story problem, at its best, we've always considered a very special--and difficult--literary form.

Nineteenth century Arithmetics and Algebras routinely incorporated a treasure trove of early story problems, indeed frequently featured them in extended groupings of miscellaneous or "promiscuous" exercises in which the harder story problems all but stole the show. Many of the very best of these story problems derived from competitive examinations, persisting in school and college textbooks from author (or "compiler") to author and from decade to decade. Such examination questions recall Elementary Algebra, even Arithmetic, as college subjects. Fractions, rates, and proportions! Linear relations, simultaneous equations, factorable quadratics! Such, in the main, was the mathematics to be brought to bear on such test questions. Elementary they were, but seldom easy!

Twelve of our favorite Victorian-era story problems follow, carefully chosen for their diversity, challenge, and lasting human interest. These particular questions have been selected for presentation at Oklahoma Convention and subsequent sharing with chapters ... from thousands of such story problems we've collected and filed over the years, from textbooks and other sources, North American and overseas.

Answers, as provided in the early schoolbooks, have been given at the conclusion of this presentation. Use them as you will! Required, of course, are full mathematical "solutions," building upon accepted and appropriate arithmetic, algebraic, and geometric insights and methodology and arriving inevitably at these numerical results. No "trial and error." No assuming "solutions in integers." Peeking at answers no doubt was resorted to by Victorian-era students--and schoolmasters--so perhaps shouldn't be wholly ruled out. Realize, however, that virtually all these story problems had their origin on competitive college examinations, time being limited and no "right answers" being around.

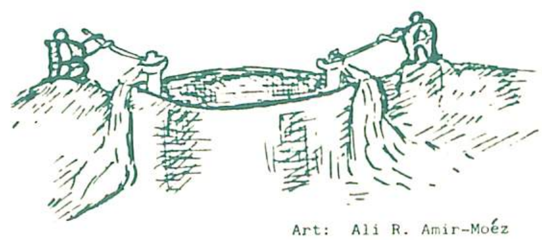
There follow our twelve favorite such story problems. Comment on solution is invited and encouraged. Interesting and unusual insights certainly will be shared. So set aside the hand-held calculator, ball-point pen, and other latter-day contrivances (not really!), transport yourself to a more leisurely, perhaps more reflective, bygone era, and try--really try!--for a first-time-around "12 out of 12."

I.
On the Road to London

A and B travelled on the same road, and at the same rate, from Huntingdon to London. At the 50th mile stone from London A overtook a drove of geese, which were proceeding at the rate of three miles in two hours; and two hours afterwards he met a stage waggon which was proceeding at the rate of nine miles in four hours. B overtook the same drove of geese at the 45th mile stone, and met the same waggon exactly 40 minutes before he came to the 31st mile stone. Where was B when A reached London?

* * *

II.
The Hold is Cleared



Art: Ali R. Amir-Moéz

The hold of a vessel partly full of water (which is uniformly increased by a leak), is furnished with two pumps worked by A and B, of whom A takes three strokes to two of B's; but four of B's throw out as much water as five of A's. Now, B works for the time in which A alone would have emptied the hold; A then pumps out the remainder, and the hold is cleared in 13 hours 20 minutes. Had they worked together, the hold would have been emptied in 3 hours 45 minutes, and A would have pumped out 100 gallons more than he did. Required the quantity of water in the hold at first, and the horary influx at the leak.

* * *

(Continued on page 3)

The Mathematical Log

ISSN 0025-5580

The Mathematical Log, with Mathematical Tall Timbers, a supplement for Chapters, is the official publication of the National High School and Junior College Mathematics Club, Mu Alpha Theta (M A Th). Mu Alpha Theta is sponsored by the Mathematical Association of America and the National Council of Teachers of Mathematics. The Log is published four times each year, in February, April, October, and December. Editorial deadline is two months prior to publication. Correspondence should be addressed to Mu Alpha Theta, 601 Elm Ave., Rm. 423, University of Oklahoma, Norman OK 73019.

Editorially Logged

The Convention—Back to School Issues of The Mathematical Log and Mathematical Tall Timbers (our Chapter supplement) are a special challenge and special pleasure for us—since we can be on the scene (this year in Oklahoma) when so many of you receive and can respond to your issues. We value the dialogue of Convention time, and would argue that it should and can be year-long and involve even more sponsors and members. Let's hear from you soon ... your priorities and your hopes for Mu Alpha Theta and for your Log.

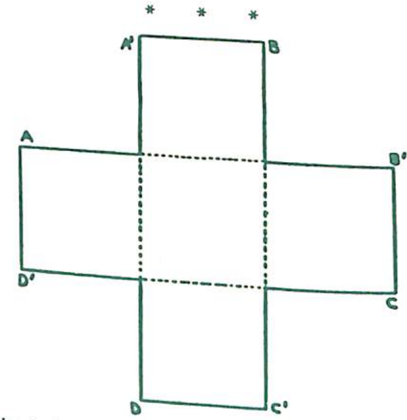
diaLogue with the editor

The Texas Mu Alpha Theta Variable, Texas State's highly readable newsletter-plus, provided us with one of our more pleasurable mental workouts of the past year, and its challenge problem—new to us—seems one well worth sharing. To quote the challenge directly: "Using six ones, how many numbers between 1 and 100 can you write? You may use any order of operation, but you must use all six ones and no more than six." Giving ourselves a 24-hour time limit, we ruled out factorials (as not elementary, and not really needed), and used addition, subtraction, multiplication, division, brackets, surds, decimals, and repeating decimals, to represent (with exactly six ones) each of the first hundred numbers, then to extend our listing through 141. We delighted in such "elegant" (well, highly symmetrical) representations as

$$37 = \frac{111}{1+1+1} \quad \text{and} \quad 101 = \frac{1111}{11},$$

but were, and still are, stopped (somehow) by 142. Should you do better, and you well may, Texas State would, we're sure, be glad to hear from you, in care of Blessed Sacrament Academy, 1135 Mission Rd., San Antonio, TX 78210.

* * *



The third dimension, we find, "adds a dimension" to a mathematical problem in both a figurative and a literal sense. Featuring "a topless cubical box," an attractive little "space" question appeared recently in our favorite elementary "problem" department, that in School Science and Mathematics (Problem 3898, proposed March 1982, solved February 1983). The problem is by one of America's top "elementary problem" people, Charles W. Trigg, of San Diego. Reproduced with permission of SSM's editors, Trigg's "topless cubical box" challenge reads as follows:

"A topless cubical box with edges of length 1 has its vertical faces hinged to the horizontal square bottom. The top coinciding vertices of adjacent vertical faces (A and A', B and B', C and C', D and D' in the accompanying figure) are connected with pieces of string of length 1. The vertical faces are permitted to fall outward until stopped by the strings. What angles do the faces then make with the surface upon which the box rests?"

Problem solving, at the right challenge level, is both satisfying and good fun. SSM welcomes student solutions to the original problems which it includes in each issue.

* * *

(Continued on page 6)

Pappus Theorem of Antiquity Yields Instructive Insights

By Ali R. Amir-Moéz
Mathematics Editor

Pappus (Παππος), the famous Greek geometer of the third century (possibly first century) who lived in Alexandria, wrote his Mathematics collections, from which the last six books out of eight come down to us [1]. Among his works one finds a formula for the volume inside of certain surfaces of revolution. This was rediscovered or possibly reproved by Guldin. In this note we would like to experiment with Pappus' formula and verify it through other formulas.

Pappus' Theorem:

Let A be the area of a plane region P enclosed in a closed curve, and l a straight line in the plane of the region which is on one side of P (Fig. 1). Suppose

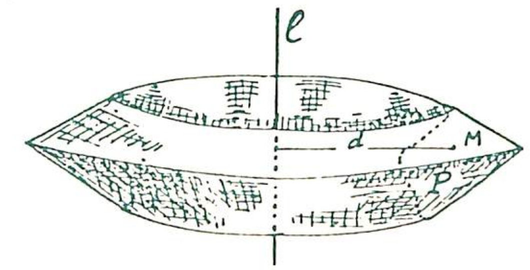


Figure 1

M is the centroid of P and d is the distance from M to l. Rotating the region about l, we obtain a solid. The volume V of this solid is A multiplied by the circumference of the circle that M traces about l. Thus we get

$$V = A(2\pi d). \qquad (1)$$

One may call (1) Pappus' formula.

Cylinders:

Let ABCD be a rectangle of dimensions a and b (Fig 2). It is clear that M, the centroid of it, is the

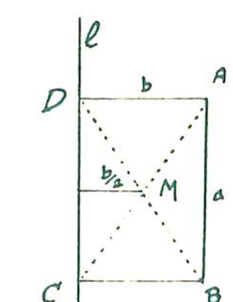


Figure 2

point of intersection of the diagonals. Now let us rotate the rectangle about l = DC. It is clear that the volume of the generated cylinder is

$$V = \pi b^2 a.$$

Next we apply Pappus' formula. It is apparent that the distance from M to l is $\frac{b}{2}$; that is, $d = \frac{b}{2}$.

Applying (1) we obtain

$$V = (ab)\,(2\pi)\,(\tfrac{b}{2}) = \pi b^2 a.$$

Now let us move l to a distance p parallel to DC (Fig. 3). Rotating ABCD about l, we obtain a cylin-

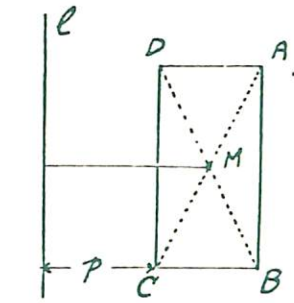


Figure 3

drical shell. Here again we apply (1), where A = ab, and $d = \frac{b}{2} + p$. So we get

$$V = (ab)\left[2\pi(\tfrac{b}{2} + p)\right] = \pi ab(b + 2p). \qquad (2)$$

Indeed, one may obtain this volume by subtracting the volume of the cylinder of radius p from the volume of the large cylinder of radius p + b, i.e.,

$$V = \pi(p + b)^2 a - \pi p^2 a. \qquad (3)$$

One can easily carry out the algebra and show that (3) is the same as (2).

Cones:

Let ABC be a right triangle, where B is the vertex of the right angle (Fig. 4). We rotate ABC about l = BC. Let M be the centroid.

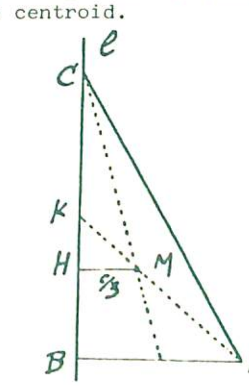


Figure 4

It is well known that M is the point of intersection of the three medians. Let AK be the median of the side BC. Then

$$AM = \frac{2}{3}AK, \quad \text{and } MK = \frac{1}{3}AK.$$

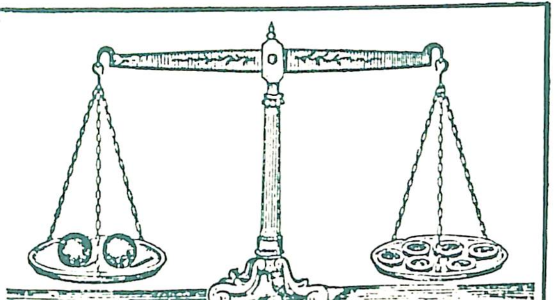
Indeed, a proof can be found in any book on elementary geometry. Now suppose MH = d is the distance from M to l, = BC. One observes that the triangles KMH and KAB are similar. Therefore

$$\frac{MH}{AB} = \frac{MK}{AK} = \frac{1}{3}.$$

(Continued on page 6)

ctorian Problems

M PAGE THREE



have cost only 54 pounds more than the expense of the sewer. Required the lengths of the streets and the sewer, and the distance of its mouth from the bridge B.

* * *

Bonus Question--for Sponsors

The Income of a Schoolmaster

The income of a schoolmaster arises partly from ten pupils residing in his house; and partly from an endowment, which produces a certain number of quarters of wheat each year. When wheat sells for 60 shillings the expenditure of his family (249 pounds) is less than his savings by a number, which when divided by twice the number of his pupils expresses the propor-

Pappus Theorem of Antiquity Yields Instructive Insights

By Ali R. Amir-Moéz
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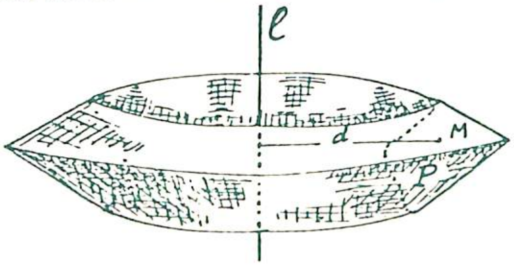

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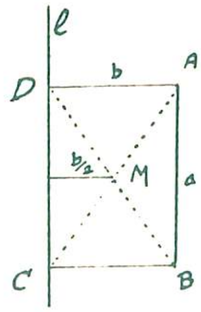

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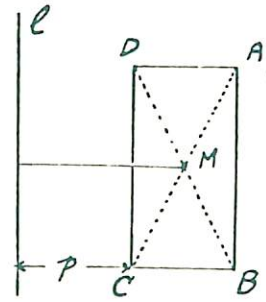

Figure 3

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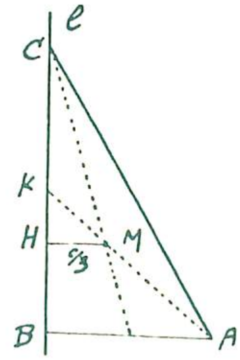

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$$\frac{MH}{AB} = \frac{MK}{AK} = \frac{1}{3}.$$

(Continued on page 6)

Pappus Theorem

FROM PAGE FIVE

This implies that

$$d = MH = \frac{1}{3}(AB) = \frac{c}{3}.$$

Rotating the triangle ABC about BC, we obtain a cone. Now we shall apply (1). Note that $A = \frac{ac}{2}$, and $d = \frac{c}{3}$. So the volume will be

$$V = \left(\frac{ac}{2}\right)\left[2\pi\left(\frac{c}{3}\right)\right] = \frac{\pi c^2 a}{3}.$$

One may experiment with other flat objects; for example, rotating a triangle about one of its sides.

Tori:

A torus provides a good illustration of the application of Pappus' formula. Let l be a line in the plane of a circle of center M and radius a (Fig. 5). Let d be the distance from M to l such that $d \gtrless a$.

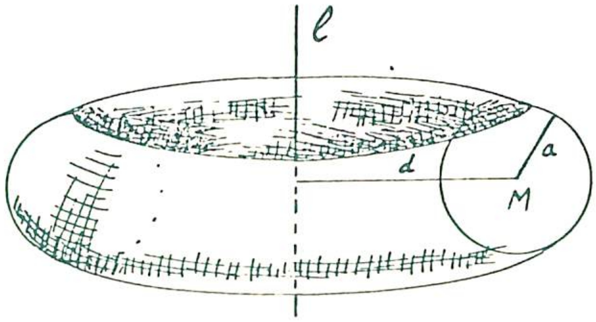

Figure 5

Then, applying (1), the volume of the torus obtained by rotating the circle about l will be

$$V = (\pi a^2)(2\pi d) = 2\pi^2 a^2 d.$$

One may, as a further example, rotate an ellipse about a line. This gives the opportunity to see that πab is the area of an ellipse whose half major and half minor axes are respectively a and b.

Obtaining Centroids:

Sometimes it will be useful to employ Pappus' formula to obtain the centroid. We shall give an example. Consider a half circle of radius a with diameter AB (Fig. 6). Let M be the centroid and d the distance

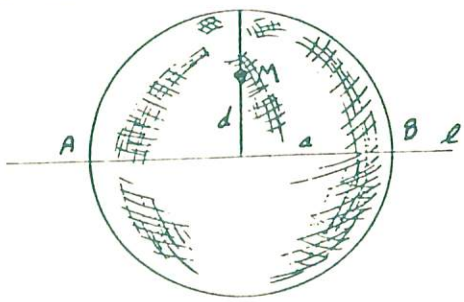

Figure 6

from M to AB. If we rotate the half circle about AB, we obtain a sphere of radius a. The volume can be written in two ways; by the use of the centroid, or by reference to the formula for the volume of a sphere of radius a; i.e., $\frac{4\pi a^3}{3}$ or by (1). Thus

$$V = \frac{4}{3}\pi a^3 = (2\pi d)\left(\frac{\pi a^2}{2}\right). \qquad (4)$$

From (4) we obtain

$$d = \frac{4a}{3\pi}.$$

This means that the centroid M of this semicircle is on the perpendicular bisector of AB, where the distance from M to AB is $\frac{4a}{3\pi}$.

Reference

[1] David Eugene Smith, History of Mathematics, 1925. Vol. I. Reprinted by Dover Publications, Inc.

diaLOGue . . .

FROM PAGE TWO

The long-division "restoration challenge" (April Log, p. 4) provided an interesting exercise in logic (by our pencil and scrap paper approach), or else a good workout for programmer and computer. Stephen Malinak, Grade X student at St. Vincent-St. Mary's High School, opted for the latter approach ... using his computer to multiply 1000 number pairs to come up with the unique result. "Computers are wonderful, aren't they!" exclaims Stephen. Yes, in their place ... and we also like Stephen's "smiling face" typewriter key.

```
234)45678(195
    234
    2227
    2106
     1218
     1170
       48
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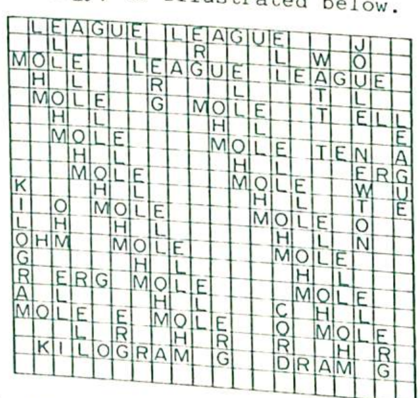
* * *

"Crossword Challenges"? Nothing seems to bring out the competitiveness, resourcefulness, and downright ingenuity of Loggers to quite the extent of these mathword puzzlers, and "Mingled Measures," our February challenge, was no exception. Brian Schar of Pulaski High School chapter, Pulaski, WI, submitted 297 points of intricately interwoven metric and pre-metric terminology, as illustrated below. Will Brian's



297 record stand? Frankly, we don't know. The clever "staircasing" and overlapping are strong points in its favor,-- but (i) previous winners have been more highly symmetrical, (ii) the all-blank 20th row may leave room for improvement, (iii) the proportion is fairly high (34/73 = 46+%) of older, 3-point terms. So it's Logmaster Brian (congratulations!), with the "ongoing" challenge: can you (or, collectively, your chapter) better Brian's impressive 297 "Measures" score?

* * *

Sam Loyd, America's master puzzlist, would have enjoyed meeting Logmaster Daniel Paul Johnson, Bedford, PA. The Bedford chapter member took on Loyd's Columbus Puzzle (December Log), correctly surmised that the winning total would be an exact 82, and came up with eight solutions. Representatively,

$$82 = 80.\dot{4}\dot{7} + .\dot{5} + .\dot{9}\dot{6}$$

The Mathematical Log

VOLUME 26, NUMBER 2 DECEMBER 1981

Giant 'Word Search' and 'Scrambled' Variants Yield Open-Ended 'Math-Word' Challenges

E	D	R	O	H	C	O	R	E	D	O	M	I	N	O	I	S	S	E	R	G	O	R	P	O
O	N	F	S	Y	M	M	E	T	R	Y	R	P	E	N	O	T	W	E	N	R	O	E	E	U
N	H	I	S	P	I	R	A	L	L	I	E	L	T	Q	U	A	D	R	A	T	I	C	U	T
O	C	B	L	O	T	N	E	G	N	A	T	A	T	E	S	S	E	R	A	C	T	I	L	N
I	N	O	S	T	E	A	C	O	P	E	R	N	I	C	U	S	W	R	L	N	Y	P	E	E
T	I	N	U	E	R	X	I	R	O	T	C	E	V	X	S	K	E	W	A	E	R	R	R	I
A	M	A	C	N	O	I	T	C	E	S	R	E	T	N	I	M	D	C	E	C	R	O	U	T
T	E	C	O	U	T	S	E	R	I	E	S	D	D	A	U	L	E	Y	R	N	O	C	T	O
U	D	C	F	S	E	E	F	U	N	C	T	I	O	N	L	S	T	W	O	E	T	A	N	U
M	I	I	L	E	M	R	R	S	Z	E	R	O	D	A	M	I	N	U	S	R	C	L	A	Q
R	A	L	U	C	I	D	N	E	P	R	E	P	X	I	V	S	R	E	M	E	A	N	N	U
E	N	T	C	U	R	V	E	N	X	E	S	I	N	I	S	I	U	D	O	F	F	E	I	P
P	O	N	E	Z	P	E	T	E	S	A	O	E	T	T	C	C	M	R	A	M	E	N	M	A
E	L	E	M	E	N	T	O	H	O	M	M	U	T	O	N	A	R	L	D	U	B	O	R	R
N	U	I	D	E	N	T	I	T	Y	N	B	P	S	R	R	I	O	I	E	C	Q	C	E	A
A	O	C	E	N	I	E	T	S	N	I	E	A	L	G	A	G	O	T	M	R	H	A	T	M
I	O	I	L	L	N	P	R	O	R	E	H	A	O	E	A	C	O	P	I	I	K	E	E	E
L	M	F	N	I	L	I	A	T	O	E	C	L	S	R	X	T	S	G	M	C	N	R	D	T
X	O	F	S	U	D	B	S	A	D	R	E	U	I	E	P	O	H	E	E	N	O	A	R	E
I	D	E	S	L	S	I	O	R	E	L	T	T	L	M	C	T	D	R	D	D	T	A	N	R
R	U	O	L	C	D	I	O	E	L	B	H	F	Y	R	T	E	M	O	N	O	G	I	R	T
T	L	C	I	I	T	N	X	A	O	M	E	S	A	B	S	P	H	E	R	E	G	E	M	C
A	U	S	O	A	L	Y	R	D	E	R	A	H	Y	P	O	T	H	E	S	I	S	O	U	T
M	S	I	R	O	T	A	N	I	M	O	N	E	D	E	D	D	E	G	R	E	E	B	S	N
A	N	G	L	E	P	R	G	I	N	T	E	G	E	R	D	A	L	O	B	R	E	P	Y	H

Competitions, anyone? How about word searches, starting with this 25 x 25 giant richly spiked with hauntingly familiar mathematical terms. Then word searches with an unusual twist, a cluster of mathematically related challenges calculated to enliven a Mu Alpha Theta chapter agenda and to keep Log correspondence coming in for months! Read on!

As an opener, reproduced above is our Giant California Mathematical Word Search, whomped up by the Editor for circulation at our UCLA 11th National Convention in August. Try this one first. In team or individual competition, it should be good for a meeting in itself. Our California Giant conceals 100 or more common mathematical words--terms, names, etc. Seek them horizontally, vertically, diagonally, forwards and backwards, but not (in this instance) "around corners." List your words, count them, then score 1 point per letter for each different word that you find. Don't list "words within words" in this first word search: in any such situations, record and score the longer word.

You may find it helpful to type or hand letter an enlarged version of the California Giant, then run off abundant "working copies" for the members of your group.

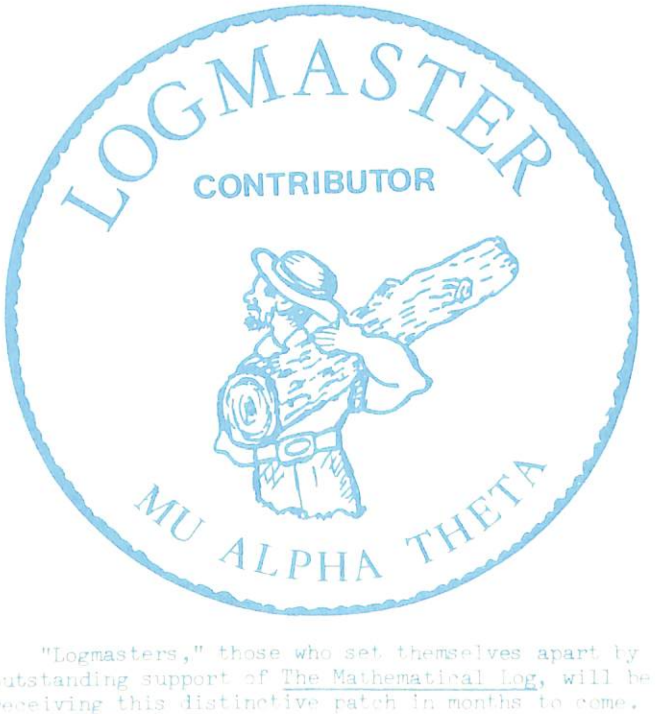
As a related and nontrivial exercise, attempt a classification of the mathematical words you find.

Develop a "system" and 100 words shouldn't be too hard to find. Remember: 1 point per letter, and the highest score wins. The Log would be interested in reporting how you made out!

So much for the traditional word search, familiar, but still fun. Now for the "spinoffs," something different, and in themselves worthy grist for the Mu Alpha Theta chapter mill.

(Continued on page 5)

GOVERNORS APPROVE SPECIAL PATCH FOR MATHEMATICAL LOG SUPPORTERS



"Logmasters," those who set themselves apart by outstanding support of The Mathematical Log, will be receiving this distinctive patch in months to come. Mu Alpha Theta Governors at their UCLA meeting gave approval to the Editor's recommendation for such recognition for writers, problem solvers, etc. Artwork is by Ali R. Amir-Moéz, himself an outstanding Log supporter.

The Mathematical Log

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KEY WEST CHAPTER READY TO SHARE UNIQUE RECIPE COMPILATION

Mu Alpha Theta chapters, by and large, frequently show two perhaps unexpected strengths: their school spirit actively involves them in the greatest diversity of projects and worthwhile events, and they're good fund-raisers for what they perceive as a worthy cause.

From Key West High School, Key West, FL, comes a report that effectively underlines both points.

Recipes from Key West and Other Places is a Mu Alpha Theta student project, with a professional cookbook publisher as printer. According to Mr. Onis Avant, chapter sponsor, the cookbook is "one of the best I've seen."

"The material is top quality, and the recipes are widespread and varied," Mr. Avant notes.

Funds raised from the project help send the club to mathematics competitions. In addition, the club sponsors scholarships and awards totalling over one thousand dollars.

Recipes from Key West may be obtained for $5.75 (postage included) from Mu Alpha Theta, Key West High School, 2100 Flagler Ave., Key West, FL 33040.

SHOPPING SPREE! A committee heads for the hobby shop to stock up on mathematical and logical games for the Chapter collection.

EDITORIAL REFLECTIONS

A collective "You're most welcome!" to the scores of thoughtful Mu Alpha Theta enthusiasts who went out of their way to say "Thank you" for The Log at our big and bustling 11th National Convention in August at UCLA. We, in turn, record our gratitude to our busy National Office, our sponsors, our contributors and correspondents, and to the extra-busy Fred Hansen and Ronald Klint who coordinated our first Southern California convention. (It's St. Louis in 1982, with energetic, ambitious Akehiko Takahashi at the helm.)

We want, too, to assure you that your Log is in good health. Your Governing Council has approved four issues for 1981-82, with up to six pages. With the untimely passing of The Mathematics Student (the NCTM student publication), a victim of rising costs, the Log's new vitality takes on even greater importance. Issues will be dated December, February, April, and October, with deadlines the 1st of the previously-named month--that is, February 1st for the April issue, April 1st for the October issue, which will be published in late Spring.

Since our 1st National Convention, held at Trinity University, San Antonio in 1968, national meetings have taken place in Pennsylvania (twice), Louisiana, Wisconsin (twice), Arkansas, Iowa, Alabama, Georgia, and California. Planning these national meetings calls for considerable lead time, of course. We understand from Governing Council deliberations that queries from chapters wishing to "host" 1983 and subsequent Conventions would be welcomed at this time.

The Log is pleased to congratulate Edward Rimland, Mu Alpha Theta president at Miami Coral Park Senior High School, Miami, FL. At UCLA, Edward was named "outstanding Mu Alpha Theta member" and recipient of the Robert Kalin Award.

The Secretary's Report to Governing Council underlined the continuing sound health of Mu Alpha Theta. A record high of 19 365 new members was recorded in 1980-81. Ninety-five new chapters brought the total on the active roster to approximately 1175.

We hope to have fuller coverage of our outstanding UCLA Convention (including photographs) next issue. Plus news of plans and hopes for "the 12th" at St. Louis. Plus your submissions, especially state and chapter news you want to share.

H.D.A.

Can You Do Better?

123 MOVES BEST REPORTED SOLUTION TO LOG'S SLIDING-BLOCK CHALLENGE

The delightful ten-piece "sliding block puzzle" featured in the Spring 1981 Log can be solved--no one has had serious doubt about that. The challenge to this readily-constructed "U-Make-It" has been to find not only a solution but an optimal solution. In how few "moves" can the deceptively tricky puzzle be solved?

One hundred twenty-three moves, suggests Math-Jeunes, the French-language, Belgian student mathematics journal which originally carried the interesting "sliding block" challenge. This "best solution" is credited by Math-Jeunes to Mme Claudine Festraets. The solution, in full, is reproduced below. Log readers are invited to submit solutions that solve the puzzle in less than 123 moves.

Recall that the ten "blocks" comprised four unit squares, five 2x1 rectangles, and one 2x2 square, all within a 4x5 unit frame. Taking one unit to be 2 cm is a good scale. Fig. 1, below, shows the initial arrangement of blocks. The shaded area is empty. The object of the game is as follows: by sliding the pieces into empty spaces, one move at a time, to bring the large (2x2) square to the lower, center position, as depicted in Fig. 2.

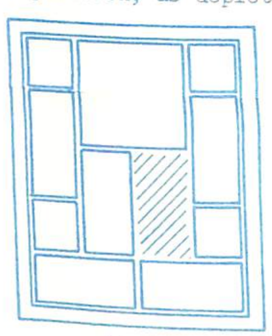


Fig. 1

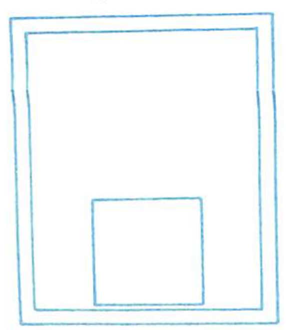


Fig. 2

A useful notation for recording moves labels the ten pieces a through j (Fig. 3): if the move is to slide d to the left, one writes d←; if, then, d is slid upwards, d↑; more compactly, these two moves can be described as d←↑. Next, if i is raised, one writes i↑; and so on.

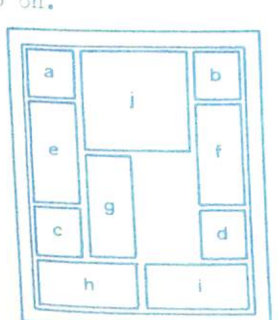
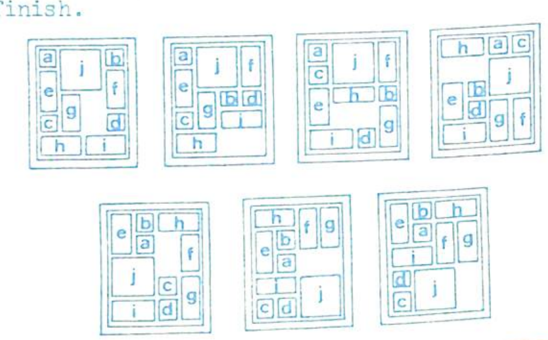


Fig. 3

The 123 moves of the Math-Jeunes "best solution" are recapitulated in that journal as follows:

1 d← 2 d↑ 3 i↑ 4 h→→ 5 c↓ 6 c→ 7 e↓↓ 8 a↓↓ 9 j←
10 b← 11 f↑ 12 d← 13 b↓↓ 14 j→ 15 a↑↑ 16 e↑↑ 17 c← 18 c↑
19 h←← 20 i↓ 21 b↓ 22 b→ 23 g← 24 c→ 25 c↑ 26 h↑ 27 i←←
28 g↓ 29 d← 30 b↑ 31 g→ 32 d↓↓ 33 b← 34 g↑ 35 d→ 36 i→
37 h→ 38 e↓↓ 39 c← 40 c↑ 41 e↑ 42 i→ 43 d← 44 g↓ 45 b→
46 h↑ 47 d↑ 48 d← 49 i→ 50 e↓ 51 h← 52 b← 53 g↑ 54 i→
55 d↓ 56 b↓ 57 b← 58 g← 59 f↓↓ 60 j→ 61 c→ 62 c↑ 63 h↑
64 b↑ 65 d↑ 66 e↑ 67 i←← 68 g↓ 69 f↓ 70 j↓ 71 c→→ 72 a→→
73 h↑ 74 b↑ 75 b← 76 j← 77 f↑↑ 78 g→ 79 d→ 80 d↓ 81 j↓
82 a↓ 83 a← 84 c← 85 c↓ 86 h→→ 87 b↑ 88 b→ 89 e↑↑ 90 j←
91 c↓↓ 92 a→ 93 b↓ 94 h← 95 f↑ 96 g↑ 97 d→ 98 c↓ 99 j→
100 e↓↓ 101 h← 102 b← 103 a← 104 f← 105 g↑↑ 106 j→ 107 a↑
108 b→ 109 e↑ 110 i↑ 111 c←← 112 d←← 113 j↓ 114 f↓ 115 g↓
116 h→→ 117 e↑ 118 b↑ 119 a↑ 120 i↑ 121 d↑ 122 d← 123 j←

As a useful check on step-by-step progress, diagrams below, from Math-Jeunes, show the puzzle at the start; after 19, 46, 73, 91, and 113 moves; and at the finish.



dia Log ue

with the editor

ONE MAN'S EXPERIENCE

TXU H LXPR LHF FHSR UX YF HMXYU UCB JBHTSTN XP BG-YUSWSUQ XI JHUCBJHUSAF: LB LBPB FSJEWQ HFDBR UX BG-YUSWSUQ...

(Continued on page 4)

DIALOGUE, FROM PAGE 3

sage. Of the 20 characters that turn up in the 105 letters of the quotation and name, 6 (I, C, A, G, H, E) account for over 60%. We've entitled this cryptogram

INTUITIVELY TRUE

ABCDACAEB (FGHI, JIFGHI, EK FGCLIFGCAMGH) LGN OIIB PKIGCHQ ERIKKGCIS. ABCDACAEB AN CLI KEEC EJ GHH NDTIKCACAEB. --IKAM CIFTHI OIHH.

* * *

Many sponsors share our view that wrestling with a good problem can be one of the better ways of promoting mathematical growth. At Mu Alpha Theta level, such problems might read like these:

Show that from any five integers, not necessarily distinct, one can always choose three of these integers whose sum is divisible by 3.

A hexagon inscribed in a circle has three consecutive sides of length a and three consecutive sides of length b. Determine the radius of the circle.

Four distinct lines L_1, L_2, L_3, L_4 are given in the plane: L_1 and L_2 are respectively parallel to L_3 and L_4. Find the locus of a point moving so that the the sum of its perpendicular distances from the four lines is constant.

Such "good problems" can be hard to come by, every sponsor knows. Recent browsing let us chance upon many hundred, however, and we're pleased to be able to share our source. 1001 Problems in High School Mathematics is being produced by the Canadian Mathematical Society in a preliminary version, with four booklets (400 problems in all, 350 solutions) currently available. Booklets are available from the Society, 577 King Edward Ave., Ottawa, Ontario, Canada K1N 6N5, at Cdn$2.50 each, postpaid. Prepayment is required.

Compilers of the collection (from which the three above questions are selected) are E. Barbeau, M. Klamkin, and W. Moser.

* * *

"We share your belief that more good students should be encouraged to work mathematical problems."

So say Bob Prielipp and N.J. Kuenzi, University of Wisconsin--Oshkosh, co-editors of very possibly the most successful "problem department" to which an interested student could turn for a challenge.

The department runs in School Science and Mathematics, and the editors were writing to give The Log permission to quote Alan Wayne's "Problem 3860," with which we'd been having particular fun.

"What is the smallest integer which can result when a five-digit positive integer (in the decimal system of notation) is divided by the sum of its digits?"

School Science and Mathematics (check with your librarian) long has encouraged submission of student solutions to the problems that it poses.

* * *

"Yes, I do my shopping at Poole, even though it is eight miles from me as the crow flies," said Beryl. "Just a matter of access."

Sam nodded. "That's why we see you so often. But aren't Alton and Bray both nearer to you?" he asked. "I know the three villages are equal distances from each other, making a triangle."

"Sure, but that's what I mean," Beryl replied. "I'm only three miles from Bray and five miles from Alton, but there's no direct road to either."

How far apart were the three villages?

Titled "No Direct Road," that "teaser" is one of exactly 100 "more difficult" puzzlers shared by prolific problemmaker J.A.H. Hunter in his Challenging Mathematical Teasers, recently released by Dover Publications, Inc. (180 Varick St., New York, NY 10014), at $2.75.

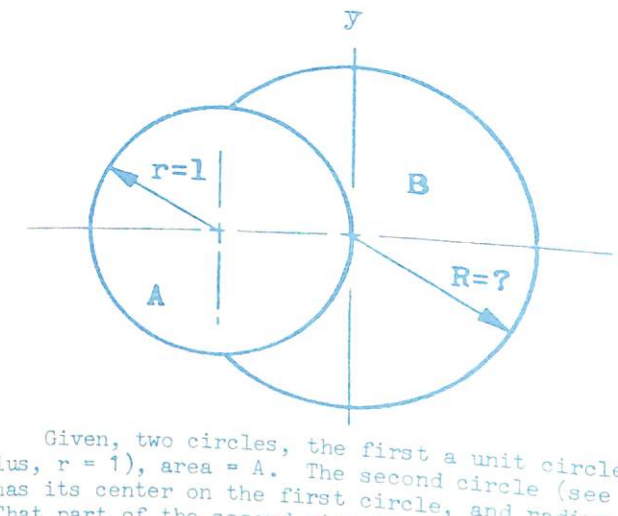
The work includes solutions to all problems, a selection of alphametics, and notes on diophantine equations and congruence arithmetic.

Perhaps representative of the 40 alphametics:

```
  C R A B B Y
  C R A B B Y
    T A B B Y
+       M A Y
-------------
S C R A T C H
```

A worthy addition to school, chapter, or personal library--and, at the Dover price, an uncommon value!

* * *



Given, two circles, the first a unit circle (radius, r = 1), area = A. The second circle (see above) has its center on the first circle, and radius = R. That part of the second circular region which lies outside the first circle has area = B.

For A = B, determine R.

The Past Masters Club, a Toronto-based group of problem solvers, has posed this and similar "simple" questions--which defy simple solutions. The Past Masters' best answer, quoted with their permission: 1.24311673443+!

Should you do better, or as well (!), the Past Masters may be contacted at Box 6427, Station "A," Toronto, Ontario M5W 1X3.

TUTORING PROGRAM PARTICULARS OFFERED BY CHAPTER

Many Mu Alpha Theta chapters pride themselves in lengthy records of school service. Tutoring programs are one of the more significant forms that such service can take.

The Mu Alpha Theta chapter at The Willows Academy (840 Vernon Ave., Glenoe, IL 60022) reports "a very active tutoring program" at their school. Interested chapters are invited to write for full particulars.

RUBIK'S CUBE VARIANTS POSE NEW CHALLENGES

CUBE POWER! As if Rubik's Cube weren't enough (see Fall Log), a host of improbable variants now offer new, if related, challenges. Here Jeanie MacEachern of Port Hood, Cape Breton, considers three from the Editor's growing collection: left to right, a 14-faced, irregular polyhedron (2 square faces, 4 3x1 rectangles, 8 isosceles trapezoids), a cuboctahedron, and a truncated tetrahedron! Cube enthusiasts may want to contact Mu Alpha Theta member Tom Bress, 417 Canyon Dr. S., Lehigh, FL 33936: Tom, who brought his Cube fascination with him to UCLA, may have set something of a Mu Alpha Theta record with a recent Cube solution in 72 seconds. Writes Tom, "That's my record: usually it takes me a minute and a half." Other Cube variants include a distinctive barrel shape (especially good for beginners), and key-ring size miniature Cubes and Barrels.

'MATH-WORD' CHALLENGES, FROM PAGE 1

So much for the traditional word search, familiar, but still fun. Now for the "spinoffs," something different, and in themselves worthy grist for the Mu Alpha Theta chapter mill.

Think about it! "Horizontally, vertically, diagonally, forwards and backwards" are traditional "search" rules, but "words" might go in spirals, or change direction at will! And letter grids might be triangular, hexagonal, or even three-dimensional! But not this issue. We'll limit ourselves to relatively small, rectangular (including square) arrays, but construct them of mathematical words, and allow hidden words (not necessarily mathematical) to "turn corners"! We'll set up point systems for scoring, and leave a diversity of specific challenges for chapters and individuals who'd like to try their hand.

In allowing "corner turning," The Log is adapting what puzzle author Linda Doherty has called the "scrambled word find" (Tempo Scrambled Word Find Puzzles #2--our daughter's favorite!; New York: Grosset & Dunlap, Inc., 1974). Author Doherty uses a 3 x 5 grid of apparently random letters (rich in vowels), and Log readers will begin with this size of array. Interestingly, the popular Games magazine has featured such "entwined" (its term) word searches in two contests, "Chimp-Off-The-Old-Block Contest" (March/April 1978) and "Son of Chimp" (September/October 1978), the Games allusion being to a persistent chimpanzee at a typewriter coming up with its relatively large (7 x 7) "random" arrays. In puzzles of this novel type, letters spell hidden words "up, down, diagonally and around corners" (as Linda Doherty puts it), consecutive letters "touching" (at a side or corner) and the same letter in a puzzle not being visited more than once in spelling out a particular word.

"Words," to be counted (let's agree), must be 3 or more letters in length.

Let's begin our mathematical foray into scrambled searches with the 15-letter grid that seems standard, 3 across and 5 down. We present such a grid constructed entirely of mathematical "short forms."

```
1.   S I N
     L O G
     T A N
     C O S
     M I N
```

Sine, logarithm, tangent, and cosine are, of course, well-known elementary functions; min, less common in school mathematics, gives the minimum value in a set--thus, min(3, 2, 7) = 2.

Seek "hidden words" in this elementary array. Do not, in such searches, let's agree, count the words used in construction, in this instance the 5 short forms. Do not count extensions (plurals, etc.) of the construction words, or words wholly contained within one construction word. In general, however, do (in scrambled searches) count and score separately plurals, words within words, verb forms, etc. Rules are arbitrary, and the game should be more fun this way!

Let's rule out, however, capitalized words. No Goa (the former Portugese colony) Taos (the New Mexico town), Lois, or Simon from the above grid!

Apart from the above rules and restrictions, have a good dictionary as your judge, and just about anything goes! You may "disentangle" 70, 80, even 90 or more words--including 8- and 9-letter mathematical terms. At 1 point per letter (let's agree), totals in excess of 400 points should be attainable. Have fun!

Apart from the above rules and restrictions, have a good dictionary as your judge, and just about anything goes! You may "disentangle" 70, 80, even 90 or more words--including 8- and 9-letter mathematical terms. At 1 point per letter (let's agree), totals in excess of 400 points should be attainable. Have fun!

Now, take our "scrambled words" activity one giant step further. Here is a sample of what can be done, a 3 x 5 grid constructed entirely of 3-letter mathematical words.

```
2.   S E T
     O N E
     S U M
     A R E
     R O D
```

"Are," for the record, is the root word for hectare, the international unit of land measure (100 ha = 1 km^2).

Again, seek "entwined" words--including, this time, a 6-letter "academic" word, a 5-letter "criminal" term, and 4-letter mathematical and physical science terms! Count words, as before. At 1 point per letter, total your score.

Now the real challenge! Choose your own 3-letter mathematical words and/or short forms. So

(Concluded on page 6)

back Log...

Old Logs, it would seem, never die. The challenges that they offer persist to confront a new year of members. "BackLog" looks to such "old" challenges, and to worthwhile results being received from interested readers.

Our Fall Log presented impressive results by Mike Purcell and Terry Bridgman in our mathematical names "'Crosswords' Challenge"--but with the observation that "we're not sure we've heard the last word." What may well be "the last word" on the subject comes from Mark Stapel, Knoxville, TN. Mike claims a 300 total, with the following as his winning solution. Note the symmetry.

F	I	B	O	N	A	C	C	I		H	E	R	O	N			Z		
I				E						E				E			E		F
B				W				H		R		N	E	W	T	O	N		I
O				T			Z	E	N	O		E		T			O		B
N	E	W	T	O	N			R		N	E	W	T	O	N				O
A				N	E	W	T	O	N			T		N	E	W	T	O	N
C					W			N	E	W	T	O	N		W				A
C			Z		T				W			N	E	W	T	O	N		C
I		H	E	R	O	N			T				W		O		E		C
			N		N	E	W	T	O	N			T		N		W		I
H	E	R	O	N		W			N	E	W	T	O	N			T		
E				E		T				W			N	E	W	T	O	N	
R		N	E	W	T	O	N			T				W			N		
O		E		T		N	E	W	T	O	N			T				P	
N	E	W	T	O	N		W			N	E	W	T	O	N			T	
		T		N	E	W	T	O	N		W			N	E	W	T	O	N
		O			W		O				T				W			L	
Z	E	N	O		T		N	E	W	T	O	N			T			E	
					O						N		P	T	O	L	E	M	Y
	F	I	B	O	N	A	C	C	I						N			Y	

Mike's "300" was widely circulated at our UCLA gettogether, and no one has equalled it. The popularity of this mathematics-related "Crosswords" has led us to produce a new version for next issue.

"Representations"? Responding to the "Boxcar" challenge (Winter Log), Jim Foerster, Spring, TX, submitted representations of 1 through 112, using (in each instance) a 3, a 6, a 9, and a 2 (in any order). He acknowledged help from Tim Grose and Truman Joe, and encouragement from sponsor Mrs. Sheri Esmond.

As Jim now knows, 113 can be written using a 3, a 6, a 9, and a 2--as:

$$\sqrt{(.2)^{-6}} - 9 - 3.$$

How much higher can representations be extended? Puzzlist J.A.H. Hunter, in correspondence, suggests that the true "limit" is 141.

"Four jolly men sat down to play . . .," in our telephone dial "correspondence" (Fall Log), and Tom Bress, Riverdale Chapter, Fort Myers, FL, was first with a full solution. Tom caught the substitution of KW for QU in "quite," necessary since there is no Q on the North American phone dial. "Might have given us a little warning!" Tom exclaims.

Janine Sanwo, Kingsburg High School, Kingsburg, CA, and Tim Grose, Klein High School, Spring, TX, can be added to the growing list of solvers of the Lewis Carroll "pigpen" cryptogram.

"If you went to bed at 8:00 at night and set the alarm to get you up at 9:00 in the morning, how many hours of sleep would you get?"

One, asserts Jennifer Olson, Western Springs, IL, the most recent to complete "A Wit-Twisting Arithmetical Apptitude Test" which Dr. Betty Lichtenberg ran in The Log of Winter 1980.

THAT'S LOGIC!

ABCDEFGHGIJ, GK GD HFI IB, GD LGMND OJ; FCP GK GD HJEJ IB, GD HBQRP OJ; OQD FI GD GIC'D, GD FGC'D. DNFD'I RBMGA.

--DHJJPRJPJJ.

'MATH-WORD' CHALLENGES, FROM PAGE 5

select and so permute them as to obtain the highest possible (!) word count and the highest possible point score.

The Log would be interested in learning--and in sharing--your choice of words for your 3 x 5 and your highest total and point score.

But, when you think about it, why a 3 x 5 grid? Why not, say, a 5 x 3? To explore further, list "entanglements" that you find in the following 5 x 3 array of familiar mathematical words. At 1 point per letter, add up your score.

3. T H E T A
F O C U S
P R I M E

This total, somehow, is less impressive. Can you discover other 5-letter mathematical words that will yield even higher word counts or point scores? Let's hear from you.

Then, too, what of the possibilities that exist in the great diversity of mathematical 4-letter words? Consider 4-letter words arranged, as below, in a 4 x 4 array. This quartet conceals a 6-letter crime and a 7-letter earth science term.

4. B A S E
F O U R
M I L E
S U R D

An impressive total. But with the right choice and right ordering of 4 mathematical 4-letter words, even higher totals, no doubt, can be reached. Let us know!

One further extension, to a mathematical counterpart to Games' efforts of the typewriting chimps. Select 7 7-letter mathematical words to substitute for the chimp's apparently random 7 x 7 grid. Here's our initial effort, concealing (by our count) well over 200 "entangled" words.

5. M I N U E N D
O C T A G O N
N U M E R A L
A L G E B R A
S E G M E N T
I N V E R S E
E L L I P S E

Your Games-like challenge: list the 20 longest "entangled" words that you find in the above 7 x 7 array. Score them, at 1 point a letter. Let's hear your total score.

Now, The Log's ultimate challenge:

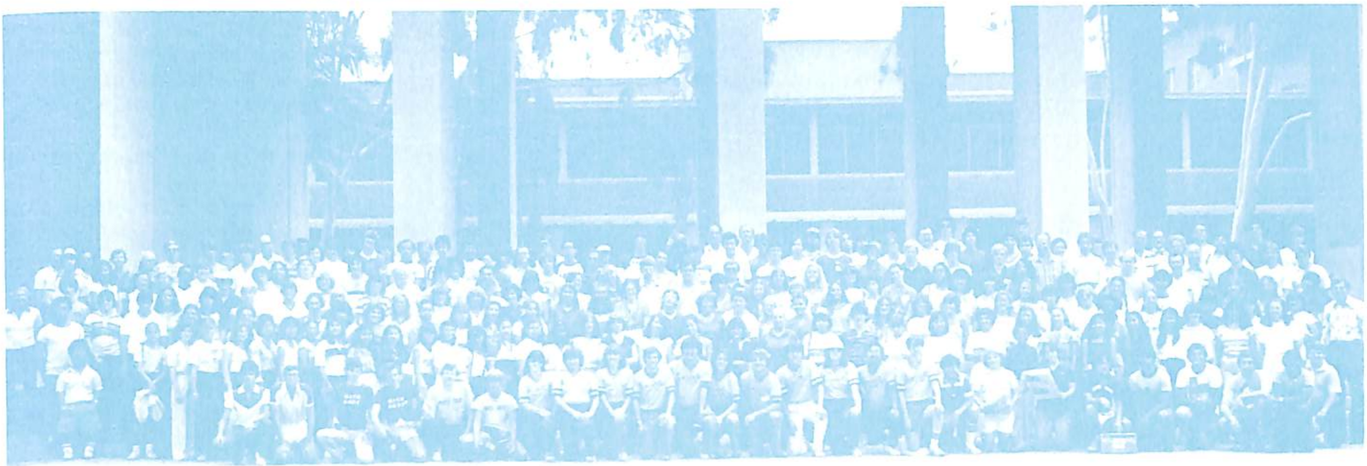
Choose your own septet of 7-letter mathematical words, and so select and so arrange them as to obtain the highest possible "count" of "entangled" words and, on the longest 20, the highest possible score.

We expect we'll be hearing from you--and sharing your findings--over the next several issues! H.D.A.

The Mathematical Log

VOLUME 26, NUMBER 3 FEBRUARY 1982

SOUTHERN CALIFORNIA, 1981



MU ALPHA THETA AT UCLA: National Conventions of Mu Alpha Theta are a high point of the school year for many chapters, faculty sponsors, and individual members. Hosted by La Canada High School and Crescenta Valley High School, the 11th National Convention (9-12 August 1981) at University of California, Los Angeles, was no exception, as these 224 (count 'em!) members and friends of Mu Alpha Theta can attest.

Geometric Methods Provide Insights Into Classical "Golden Section"

by Ali R. Amir-Moéz
Texas Tech University

Studying the shapes of large rooms such as temples, etc., one observes that the ratio of the length to the width of most of these places is almost the golden ratio. It isn't certain whether this fact has been intentional or by chance this has become pleasing to the eye. Research in archeology and related subjects suggests that the golden ratio has intentionally been used. For example, the ratio of lower edge to the altitude of pyramids is approximately the golden ratio.

Whatever we present here pertains to the mathematical aspects of the golden section.

1. The Golden Section: Let AB be a line segment and C a point on AB such that

$$\frac{AB}{AC} = \frac{AC}{CB}.$$

Fig. 1

Then we say C divides AB into golden section (Fig. 1). Construction of C has many solutions. We approach the problem through the geometric solution of a second-degree equation.

2. The Power of a Point with Respect to a Circle: Let (C) be a circle of center C and P a point in the plane of the circle (Fig. 2). Let a line through P intersect (C) at A and B. Then (PA)(PB) is independent of the position of the line. (Here PA and PB are directed segments.) The constant value of (PA)(PB) is called the power of P with respect to (C). The diagram contains two parts; for one P is outside and for the other P is inside the circle (Fig. 2).

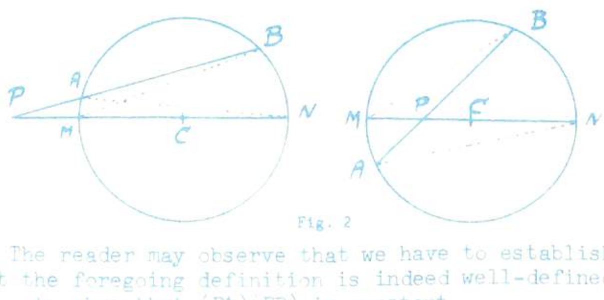


Fig. 2

The reader may observe that we have to establish that the foregoing definition is indeed well-defined, i.e., to show that (PA)(PB) is constant.

(Continued on page 4)

The Mathematical Log

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'LOGMASTERS' NAMED

The Mathematical Log has named its first four "Logmasters," outstanding contributors to Mu Alpha Theta's unique publication for student members. To each "Logmaster" goes the distinctive patch pictured in the December Log.

Initial winners are:

Tom Bress, member, Riverdale Chapter, Fort Myers, FL, a regular contributor of solutions, items, and editorial suggestions.

[illegible] Patonai, sponsor, St. Vincent-St. Mary High School, Akron, OH, whose interest and enthusiasm has resulted in some outstanding student contributions.

Ali R. Amir-Moéz, author and artist, Texas Tech University. Dr. Amir-Moéz has been a long-time supporter of Mu Alpha Theta and The Mathematical Log, and his distinctive graphics are featured on the "Logmaster" patch.

Betty Lichtenberg, former Editor, University of South Florida. Now increasingly active in National Council of Teachers of Mathematics affairs, Dr. Lichtenberg continues her great interest in The Log and is among its most faithful correspondents.

EDITORIAL

St. Louis Calls

For all of us in Mu Alpha Theta, it's none too early to be thinking about Convention! The "think tank" in Wentzville, very clearly, has been working overtime for some months, and what they've come up with is indeed impressive. With Akehiko Takahashi enthusiastic and at the helm, Wentzville High School chapter has a varied and most colorful agenda for our 12th National Convention. The setting is to be Washington University, St. Louis, and the dates are August 9-12.

St. Louis means The Arch (a most exotic, futuristic "climb"), Six Flags, the Zoo, and much more. Convention means outstanding speakers, contests, socializing, awards. The Math Bowl, Math Relays, chess and backgammon championships, a Rubik's Cubathon, and more are on the busy agenda.

Write Akehiko Takahashi (Mathematics Department, Wentzville High School, #1 Campus Dr., Wentzville, MO 63385), and he'll provide fuller details.

Set up your own think tank and come up with a way to make the trip! See you in St. Louis--for Mu Alpha Theta's 12th National Convention, August 9-12.

H.D.A.

GLIMPSING THE INFINITE

ZYXWV TSXWR QWPX, SNVVSX TSXWR MLKJ VQXNY HWGFR VK HNVX 'XD,

WJC SNVVSX TSXWR QWPX SXRRXY TSXWR, WJC RK WC NJTNJNVMD.

WJC ZYXWV TSXWR VQXDRXSPXR NJ VMYJ, QWPX ZYXWVXY TSXWR VK ZK KJ,

BQNSX VQXRX WZWNJ QWPX ZYXWVXY RVNSS, WJC ZYXWVXY RVNSS, WJC RK KJ.

--WMZMRVMR CX DKYZWJ.

Secretary's Corner

From Secretary-Treasurer Harold Huneke, the following timely notes:

From January 1, prices for Mu Alpha Theta jewelry and banner are as follows: pin, $[illegible]; charm, $5.75; button, $[illegible]; banner, $[illegible].

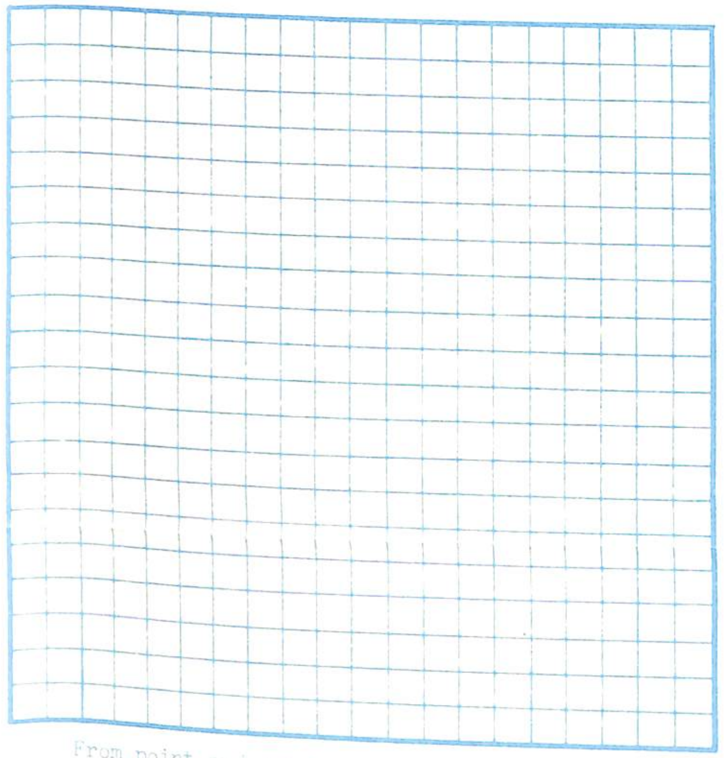
States reporting meetings last year were Florida, Mississippi, South Carolina, [illegible], Texas, and Wisconsin ... and there may have been others. Meetings had excellent programs and competitions and were well attended. Anyone interested in starting a state organization can receive information from Mu Alpha Theta's national office.

As is traditional, a [illegible] Breakfast will be held at the NCTM annual meeting--[illegible], Friday, April 1[illegible], 7 a.m. ([illegible] Inn Downtown). Reservations will be needed--send names to Harold Huneke--but actual payment (about $[illegible]) will be at the meeting.

Also, whatever you do, don't leave [illegible] without a visit to the [illegible] [illegible].)

New "Crosswords Challenge" Focuses on Geometry

The vocabulary of school Geometry is rich and diverse. What better source, then, of terminology to challenge the zealous "word packer" in another Mathematical Log mathematics-related "Crosswords Challenge"!



From point and ray to square, cube, and tesseract, listed below are, in all, 20 elementary geometric terms. After each (the name of a figure) is derived a number, a point score associated with the term and the figure--the number of letters in the word multiplied by the "dimension" of the figure. Thus, for cube, a 4-letter word for a 3-dimensional figure, the point score is 4x3, or 12.

Twenty geometric "math words" to be used in this competition, with associated point scores, are as follows:

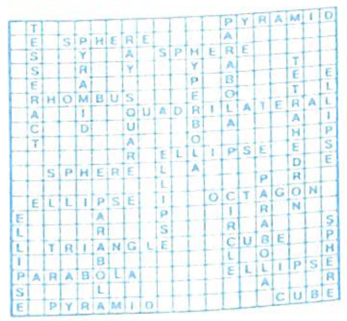
CIRCLE (6x2 = 12)	PRISM (5x3 = 15)
CONE (4x3 = 12)	PYRAMID (7x3 = 21)
CUBE (4x3 = 12)	QUADRILATERAL (13x2 = 26)
CYLINDER (8x3 = 24)	RAY (3x1 = 3)
ELLIPSE (7x2 = 14)	RHOMBUS (7x2 = 14)
HYPERBOLA (9x2 = 18)	SPHERE (6x3 = 18)
LINE (4x1 = 4)	SQUARE (6x2 = 12)
OCTAGON (7x2 = 14)	TESSERACT (9x4 = 36)
PARABOLA (8x2 = 16)	TETRAHEDRON (11x3 = 33)
POINT (5x0 = 0)	TRIANGLE (8x2 = 16)

The challenge? Take a 20x20 grid--400 unit squares. Write in "math words" from the given list, left to right or top to bottom, [illegible]. Allow words to "cross," [illegible] puzzle. Each time you [illegible] associated with that particular word. [illegible] points; three quadrilaterals, [illegible] terms need be used. A term may be used any number of times. The highest [illegible] wins. But

A "Crosswords Challenge" calling for the packing of mathematicians' names in a 20x20 grid (The Log, Winter 1981)--remember it?--proved to be among the more successful of competitive activities featured in The Log in recent years. Mark Stapel's point score of 300 in this initial competition (The Log, December 1981) stands unequalled at presstime--and could well be unbeatable. Hence the time is ripe for a new "word packing" competition. This one takes its inspiration from geometric shape and form.

note and be guided by the following restrictions, calculated to make for lively competition.

All words in your grid must be connected. There can be no "isolated" words or clusters of words. Also, words which fail to intersect have horizontal and vertical separation, adjacent squares not being filled except by letters from the same word.



Illustrated is a perhaps representative solution which meets all requirements. Using 15 geometric terms (including 6 ellipses) for 29 entries, it totals a mediocre 191 (check this). Your challenge is to do better, much better, and to let The Log and its readers know your results.

Have fun! Individually, or the whole chapter in collaboration or friendly competition, see what kind of "Geometric Challenge" record you can set. B.D.A.

MISS MISSISSIPPI 1981, Karen Hopson, 21, of Vicksburg, a senior at University of Mississippi, lists Mu Alpha Theta efforts high on her resumé of school-related accomplishments, The Log is pleased to learn. Karen was Mu Alpha Theta chapter president, class president, yearbook editor, and valedictorian at St. Aloysius High School. Winner of a $3000 scholarship award as a Miss America semifinalist and a $1500 preliminary swimsuit award, Karen hopes to obtain her doctorate in Speech Pathology and Audiology.

CLASSICAL GOLDEN SECTION, FROM PAGE 1

Proof: We draw the line PC. This line intersects (C) at M and N. We observe that triangles PMB, PAN are similar. Thus

$$\frac{PA}{PM} = \frac{PN}{PB},$$

and therefore

$$(PA)(PB) = (PM)(PN).$$

If we choose a sense on PC and let PC = d, and the radius = r, then we obtain

$$(PA)(PB) = d^2 - r^2.$$

Details are left to the reader. We may note that if P is on the circle the power of P with respect to (C) is zero.

3. A Converse Theorem: Let four points A, B, C, and D be given such that

$$(PA)(PB) = (PC)(PD),$$

where P is the point of intersection of the lines AB and CD. Then the four points A, B, C, and D are on a circle.

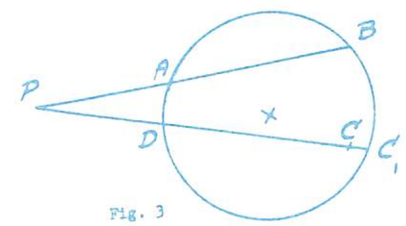


Fig. 3

Proof: Suppose A, B, and D are not collinear. Then there exists a circle through these points (Fig. 3). Suppose this circle intersects the line PD at C_1. From §2, it follows that

$$(PD)(PC_1) = (PA)(PB) = (PD)(PC).$$

Thus $PC_1 = PC$ and $C_1 = C$. Indeed, when A, B, and D are collinear the point P is not defined.

4. Geometric Solution of Quadratic Equations: Let

$$ax^2 + bx + c = 0, \quad a \neq 0$$

be a quadratic equation, where a, b, and c are real numbers. Let x_1 and x_2 be roots of this equation. Then

$$x_1 + x_2 = \frac{-b}{2a}, \quad x_1x_2 = \frac{c}{a}.$$

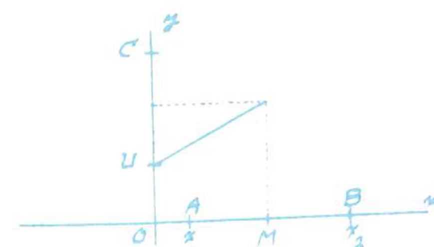


Fig. 4

We consider a rectangular coordinate system (Fig. 4). On the y-axis let U correspond to 1, and C correspond to $\frac{c}{a}$. If A and B respectively correspond to x_1 and x_2 on the x-axis, we observe that

$$(OA)(OB) = (OU)(OC) = x_1x_2.$$

Thus the four points A, B, U, and C are on a circle. We know that the center of this circle is on the perpendicular bisector of the segment UC and also on the perpendicular bisector of AB. Obviously one can easily find U and C. Now let M be the midpoint of the segment AB. Then

$$OM = \frac{1}{2}(OA + OB) = \frac{1}{2}(x_1 + x_2) = \frac{-b}{2a}.$$

> If 12 oxen eat up 3 1/2 acres of grass in 4 weeks, and 21 oxen eat up 10 acres in 9 weeks, how many oxen will eat up 24 acres in 18 weeks; the grass being at first equal on every acre, and growing uniformly?
>
> Emerson's last question, 1837

FOR OLD TIMES' SAKE! Frederick Emerson saved this tough problem for the final exercise of his classic North American Arithmetic (Boston, 1837), and with good cause. The question is the traditional "work problem" of 19th century school mathematics, but with a difference!--the difficulty rests with the 3 final words. The Editor has featured this problem on his "business card," distributing it widely at our UCLA Convention. "Model solutions" were invited: it's not too late to send yours to The Log.

Thus M can be constructed and the center of the circle through A, B, C, and U is also on the perpendicular to the x-axis at M. Let the center be D. Then DC = DU is the radius. One may discuss the different cases: If the circle intersects the x-axis, the equation has two real roots, if it is tangent there is a double root, and if the circle does not intersect the roots are complex.

5. The Equation of Golden Section: Let us consider Fig. 1 again, and let AB = 1 and AC = x. Then from the equality

$$\frac{AB}{AC} = \frac{AC}{AB}$$

one obtains

$$\frac{1}{x} = \frac{x}{1-x}$$

or

$$x^2 + x - 1 = 0.$$

The roots are

$$\frac{-1+\sqrt{5}}{2} \text{ and } \frac{-1-\sqrt{5}}{2}.$$

Indeed, we are only interested in the positive root. To construct the roots we consider a coordinate system (Fig. 5). We let OU = 1. Since

$$x_1x_2 = -1 \text{ and } \frac{1}{2}(x_1 + x_2) = \frac{-1}{2}$$

to C corresponds -1 on the y-axis and to M corresponds $\frac{-1}{2}$ on the x-axis. This way the point D, i.e., the center of the circle which gives the roots, coincides with M. Thus the circle of center M and radius MU intersects the positive part of the x-axis at K. Now we take A = 0 and choose AB = 1. Then K divides AB into the golden section.

6. An Approximation of the Golden Ratio: In Fig. 1, if we choose $\frac{AB}{AC} = y$, we obtain

$$y^2 - y - 1 = 0.$$

The positive root of this equation is

$$y = \frac{1+\sqrt{5}}{2} \approx [illegible]$$

This ratio is close to [illegible] which is practically used for the golden ratio.

(Continued on page 5)

CLASSICAL GOLDEN SECTION, FROM PAGE 4

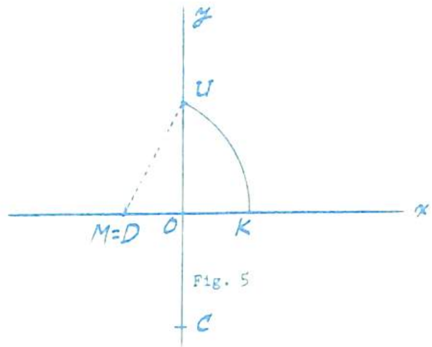


Fig. 5

6. An Approximation of the Golden Ratio: In Fig. 1, if we choose $\frac{AB}{AC} = y$, we obtain

$$y^2 - y - 1 = 0.$$

The positive root of this equation is

$$y = \frac{1 + \sqrt{5}}{2} \cong 1.618.$$

This ratio is close to $\frac{8}{5}$, which is practically used for the golden ratio.

7. The Regular Decagon: The ratio of the radius of the circumscribed circle of a regular decagon to its side is the golden ratio.

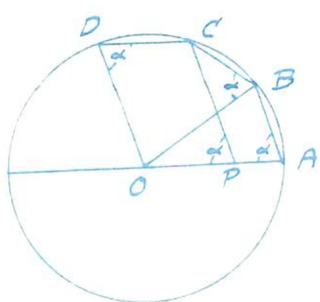


Fig. 6

Proof: Let AB be the side of a regular decagon in a circle of radius 1 (Fig. 6). We consider the three sides AB = BC = CD. Suppose that the angle OAB is α. We draw CP parallel to OD. Thus the angle OPC is also α. Therefore AB is also parallel to CP. We observe that triangles CBQ and OPQ are congruent and thus

$$QP = QB = PA.$$

If we take OA = 1 and OP = DC = AB = x, we obtain AP = PQ = QB + 1 - x. The triangles OAB and OPQ are similar and we get

$$\frac{OA}{OP} = \frac{AB}{PQ}.$$

We, thus, obtain

$$\frac{1}{x} = \frac{x}{1 - x}$$

or

$$x^2 + x - 1 = 0.$$

Consequently the construction of the side of a regular decagon is the same as the construction of the golden section. The construction we will leave to the reader (see Fig. 5).

A longtime supporter of Mu Alpha Theta and of The Log, Dr. Amir-Moéz teaches Mathematics at Texas Tech University, Box 4319, Lubbock, TX 79409.

OLYMPIAD WINNERS! Mu Alpha Theta takes pleasure in congratulating the United States team, winners in the 1981 International Mathematics Olympiad. Here the U.S. team poses in front of the statue of Albert Einstein in the National Academy of Sciences garden following the awards ceremony. Left to right, Benjamin N. Fisher, David S. Yuen, Gregg N. Patruno, Noam D. Primer, Richard A. Strong, James R. Roche, and Brian R. Hunt. The team photo first appeared in Focus, newsletter of the Mathematical Association of America.

'Buds' Seeks Authors

Mathematical Buds, Mu Alpha Theta's unique compilation of student mathematics papers, welcomes submissions at any time, accepted papers being scheduled for the next available Buds collection.

Queries and submissions should be directed to the Editor-Coordinator of Mathematical Buds, Harry D. Ruderman, 262[illegible] Davidson Ave., Bronx, NY 10468.

Submitted papers normally will have won a top award at a Math Fair held in the student's region. Where no Math Fair is held, the paper must have the sponsorship of a Mathematics teacher.

Four good copies of the paper must be submitted so that judges may read the paper simultaneously.

Submitted papers should include (1) name, address and telephone number of the student author, the school, and sponsoring teacher; (2) title and short abstract; (3) what prompted the author to write the paper; (4) a short blurb about the author: hobbies, aspirations, etc.; (5) credits, if any.

Papers must be readable by a reasonably capable junior or senior high school student. (Many papers have been rejected because they lack this important quality.) Papers with some originality are preferred.

Papers should be accompanied by a stamped self-addressed envelope so that they may be returned if unacceptable.

ELECTRIFYING!

BCDFD GH JKBCGJL HK MFKNGPGQ GJ RBGN-GBGDH SH STHBFSQBGKJH.

--PSFSVSW.

diaLogue

with the editor

"The principal had the students in a school line up on the football field in rows of ten abreast for the flag ceremony. When it was done, it was noticed that there were only nine students in the last row. 'This won't do at all,' said the principal. 'Line up in rows of nine, then.' When this was done, the last row had only eight students. 'The president of the school board isn't going to like this,' the principal declared. 'Line up eight abreast.' Of course, there were only seven students in the last row. At this point the mathematics teacher told the principal that it was hopeless to continue since the number of students N always left a remainder of $k - 1$ when divided by k for $k = 2, 3, 4, \ldots, 10$. What is the smallest number of students that could have been on the field?"

Vicki Schell of Jefferson City, MO posed that delightful challenge in the always interesting "Problem Department" of School Science and Mathematics (Problem 3866, October 1981). Department editors N. J. Kuenzi and Bob Prielipp, who encourage submission of student solutions, have given The Log permission to share what we see as a fine illustration of what once was called "higher arithmetic"--number theory, where some of the best "problem solving" in elementary mathematics is to be found.

Seek out good problems, and share them. Genuine problem solving is the best fun there is in connection with mathematics learning.

* * *

Said Robin to Richard, "If ever I come
To the age that you are, brother mine,
Our ages united would amount to the sum
Of years making ninety-nine."
Said Richard to Robin, "That's certain, and if it be fair
For us to look forward so far,
I then shall be double the age that you were
When I was the age that you are."

Old mathematics books hold, for me, a special fascination. This old Algebra problem in verse form derives from one such book, Mathematical Wrinkles, written and published by one S.I. Jones in 1912 and 1923 and located by me in a second-hand bookstore in our 1982 convention city, St. Louis. You may want to try your hand at Jones' old verse problem; or this question, which Jones classes as Arithmetic:

"The hour, minute, and second hands of a clock turn on the same center. At what time after 12 o'clock is the hour hand midway between the other two? The second hand midway between the other two? The minute hand midway between the other two?"

The perhaps most interesting problem classification in Jones' Wrinkles is Mathematical Recreations. Here one finds a remarkable variant of the Spider and Fly classic ("diaLogue," Spring 1981), worded essentially as follows:

"On a suspended block of glass, 10 inches long, 4 inches wide, and 4 inches high is a spider and four gnats. The spider is on one end 1/3 inch from the bottom and midway between the sides. The gnats are on the other end. Three of them are 1/3 inch, 2/3 inch, and 1 inch, respectively, from the top and midway between the sides. The fourth is 1 30/37 inches from the top and on an edge.

"Determine the shortest path possible, by way of the six faces of the block of glass, for the spider to catch the four gnats and return from the place from which he started."

Jones' answer (blending fractions and surds in a four-place decimal approximation): 27.3083 inches. Which agrees most satisfyingly with our exact expression,

$$14 + (\sqrt{1\,531\,834} + 6\sqrt{1594})/111.$$

As an aid to visualization, a cardboard model (say a shoebox) really helps.

* * *

Tom, Dick, and Harry (let's say) visited the corner candy store, and found an interesting contest.

Chocolate bar wrappers read, "One in 24 chances to win one of 3 million bottles of [soft drink] ... Three million chances to win are randomly inserted in 72 million specially marked [candy bars]." (This part is true. This particular "contest" was in full swing across Canada at press time.)

Observed Tom, "With all those 'specially marked bars,' I'd have to buy literally millions to be absolutely certain of a win. I'd have to buy thousands to be even 99.9% certain."

In his second statement, Tom was wrong.

Observed Dick, "There are 16 'specially marked bars' in this box. If I bought them all, I wonder how many drinks I'd win. I'd have an at least fifty-fifty chance of at least one winner, I'm sure."

Dick was wrong.

Observed Harry, "I've found 23 of this kind of 'specially marked bar.' If I opened them all I'd be more likely to find exactly one winner than any other of the possible results, say two winners or none."

Harry, too, was wrong.

"Probability" questions can be challenging--and tricky, especially if intuition is unduly relied upon. Investigate at leisure the statements of our Tom, Dick, and Harry. Consider other aspects of this interesting problem situation.

We'll "open up" (perhaps with your help) this and similar "odds" considerations in a future Log.

* * *

Those keeping busy with "mathematical scrambled word searches" (December Log) may find challenge or consolation in our own totals to press time. For #1, SIN/LOG/TAN/COS/MIN, 95 words (including ICOSAGON, a 20-sided polygon, and MISALIGN), for 423 points. For #2, SET/ONE/SUM/ARE/ROD, 45 words, 167 points. For #3, THETA/FOCUS/PRIME, 38 words, 142 points. For #4, BASE/FOUR/MILE/SURD, 95 words (including BOULDERS), 443 points. Big #5, MINUEND/OCTAGON/NUMERAL/ALGEBRA/SEGMENT/INVERSE/ELLIPSE, yields 340 words (!), including ABROGATION, MEMBRANES, RATIONALE, for 168 points (scored from 20 longest "entangled words"). We'll be interested in your top scores and improved word grids. Have fun!

Photo Available

Members and sponsors interested in obtaining prints of the Mu Alpha Theta UCLA Convention group photograph may contact our Convention photographer, Frank Moody. Frank offers an 8x10 for $3, postpaid. Write Moody Photography, 13854½ Chase St., Panorama City, CA 91402.

Precedent-Setting Demonstrations

$$\log_5 6 = e, \quad \log_5 3 = \frac{i}{2}\,!$$

YOUR DATE IN ST. LOUIS ... AUGUST 9-12.

--SEE PAGE 4

The Mathematical Log

VOLUME 26, NUMBER 4 APRIL 1982

Algebraic Manipulation Confirms You're Older than You Think

One wonderful old book of mathematical insights (see "diaLOGue") provides this algebraic demonstration that (in case you doubted it) you are as old as Methuselah.

Let the number of years in Methuselah's age be x, the number of years in your age be y, and the sum of these numbers be s.

Then

$$x + y = s$$

Multiplying both expressions,

$$(x + y)(x - y) = s(x - y)$$

$$\therefore x^2 - y^2 = sx - sy$$

Rearranging,

$$x^2 - sx = y^2 - sy$$

"Completing squares,"

$$x^2 - sx + \frac{s^2}{4} = y^2 - sy + \frac{s^2}{4}$$

$$\therefore (x - \frac{s}{2})^2 = (y - \frac{s}{2})^2$$

$$\therefore x - \frac{s}{2} = y - \frac{s}{2}$$

Adding half the sum,

$$x = y$$

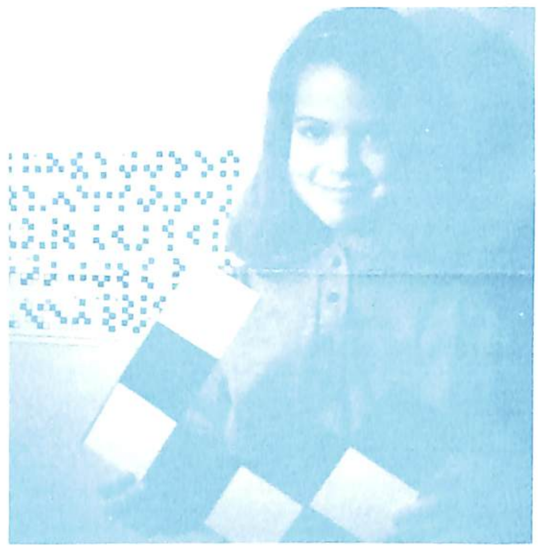
Hence, x, the number of years in Methuselah's age, equals y, the number of years in your age ... so, undeniably, you are as old as Methuselah!

INVERSE OPERATION

I7MAU5NA43E 4I AR8 5SY7K Y88K4E'' C37 ZE3S K8II A3X5C AR5E C37 X4X C8IA8UX5C.

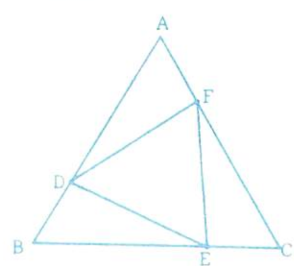
--68668U24EA 65AAC

The reference is literary, of course--from the contemporary North American author, NR5UK8I INR7KA9. 68668U24EA 65AAC is the heroine of his classic, 685-E7AI. Adapted (enciphered) from Mathematical Digest.



WOULD YOU BELIEVE 56.2 seconds to assemble this fifty-piece dissection of a 10x10 checkerboard into asymmetrical "octominoes"--the challenge puzzle in the Spring 1981 Log? You would?--then turn to the 50 pieces as depicted on page 4--and let us know your "best time."

60 SECOND CHALLENGE:



Try this, in 60 seconds! DEF is an equilateral triangle. Also, in the figure, AF = BD = CE. Prove that ABC is equilateral. Then see April Editorial.

The Mathematical Log

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The official publication of the National High School and Junior College Mathematics Club, Mu Alpha Theta, which is sponsored by the Mathematical Association of America and the National Council of Teachers of Mathematics. Address correspondence to: Mu Alpha Theta, 601 Elm Avenue, Room 423, The University of Oklahoma, Norman OK 73019.

EDITORIAL

On April Logging...

Martin Gardner, admired and respected as a popularizer and interpreter of mathematics, did it, and particularly well, a couple of Aprils ago. Pointing out, with justification, the general slowness--the time lag--in getting results into print in present-day mathematics and the sciences, Gardner devoted an entire "Mathematical Games" section in the prestigious Scientific American to preliminary reporting of staggering breakthroughs and discoveries in diverse branches of mathematics and in related fields. The whole thing was, of course, one glorious hoax, an April Fool intellectual extravaganza, but many who might have known better apparently were "taken in." Mathematicians and scientists were quick to decry flaws and inadequacies, needless to say--but, conspicuously, primarily those in the context of their particular specialty or discipline. In the spirit of Gardner, then, The Mathematical Log's April front page this issue is rich in nonsense and delusions, a happy digression for those of us who suspect that life, even mathematics, occasionally can be too serious to be fully enjoyed.

No, Virginia, $\log_5 6$ is not e (pure nonsense--see page 4), you're not as old as Methuselah (at least not by Jones' delighful "proof," page 1), and 7-year-olds (depicted, page 1, our puzzle-fan daughter, Rosalie) do not master problems ("in 56.2 seconds" or otherwise) that no one within Mu Alpha Theta has yet reported being able to solve!

However, in some ways there's more to all this than pure if irreverent fun. Such "paradoxes" as our "Old as Methuselah" demonstration are instructive as well as pleasurable, for they sharpen our ability to spot a mathematically inadmissible step. Commonly, in Algebra or Arithmetic, the inherent fallacy (however disguised) will involve division by zero, as in 2x0 = 1x0 (which is true), therefore 2 = 1, or else equating of principal and non-principal roots, as in 9 = 9 (true), therefore taking roots +3 = -3. Geometric paradoxes are instructive, too, their fallacy commonly stemming from diagram misinterpretation. This issue exhausts in 4 pages our page quota for Volume 26 Logs, but a subsequent, larger, issue will permit "demonstration" that every triangle is, indeed, isosceles.

That "60-second challenge," the little triangle to be proven equilateral, is perhaps the wickedest April Fool item on the page. It looks easy. It's not! Give yourself a bit more than 60 seconds: you're in for a good geometric workout, and very possibly a satisfying session of challenging mathematical proof.

H.D.A.

PUBLISHED WITH PRIDE. Reproducing a newsphoto from a North Carolina hometown paper, The Log depicts Mathematics Club president John Hargette, vice president Michelle Jones, secretary Tina Setzer, and treasurer Maria Medford with a preview copy of the Western Carolina University Mathematics Contest Book, a Mu Alpha Theta project at Tuscola Senior High School. Copies of the ten-year compilation of contest questions and solutions may be ordered from the Waynesville chapter.

ORDERING CAROLINA CONTEST BOOK

"A cooperative effort," Mu Alpha Theta compiling questions, working solutions, and indexing, and the Business Department doing the printing. That's how John A. Goebel, faculty sponsor, describes the chapter's compilation of contest questions (photo above).

(Continued on page 4)

Further 'Mathword' Groupings Extend 'Scrambled' Search

Introduced to Mu Alpha Theta (as math-related fun) at UCLA Convention and through the December 1981 Log, mathematical "scrambled word searches" (so-called) take place within rectangular (including square) arrays of the letters of carefully selected mathematical words. "Horizontally, vertically, diagonally, forwards or backwards, even changing direction," such "searches" have aroused some healthy interest at chapter level, sponsors report. "Ground rules" are spelled out in the December Log. Something of a knack is called for in constructing really good "search" grids, however. As "grist for the mill" of interested chapters (or further fun for interested individuals), The Log, accordingly, offers herewith an additional grouping of tested "math word" patterns of assorted sizes, along with indications of solution possibilities.

Preliminary notes on the solution potential of 5 initial "scrambled word searches" were carried in "diaLogue," February 1982 Log.

A productive 3x5 grid, first published here, is the following:

```
6.  T O N
    A R C
    E L L
    D A Y
    T W O ,
```

which conceals well over 70 admissable "scrambled words" (CLEARLY, TALLER, ALERT, ...), worth (at 1 point per letter) close to 300 points. These are better by half than results reported to date for the SET/ONE/SUM/ARE/ROD December "math word" grid.

An equally impressively endowed 4x4 mathematical word grid is

```
7.  M E A N
    A R E A
    S I D E
    M O R E ,
```

also harboring more than 70 admissible "scrambled" words of 3 to 7 letters for a point score close to 300.

A really good 5x3 grid of "math words" evidently is harder to come by, but

```
8.  R A T I O
    U N I O N
    S E V E N
```

scrambles at least 53 admissible words. (We rule out EVEN, EVES, EVE, ION, RAT, and TAR as "contained" in a given word; also RATION as an "extension" of a given word. VENUS, the planet or goddess, we exclude as a proper noun. See "scrambled search" rules, December Log.)

A new 7x7 grid of mathematical terms known to conceal several hundred admissible "scrambled words" is

```
9.  T H E O R E M
    E L E M E N T
    H E C T A R E
    D E C I M A L
    I N T E G E R
    M O D U L U S
    T A N G E N T .
```

Among the longer "scrambled words," look for INDULGES, LECTERN, and GARNET. Capitalized, and therefore inadmissible (but fun to hunt) are the names of a mathematical great, a world capital, and an outlying satellite! Your report on words found and total points (the letter sum of, in this instance, the 20 longest words) would be welcomed by The Log and by your fellow searchers.

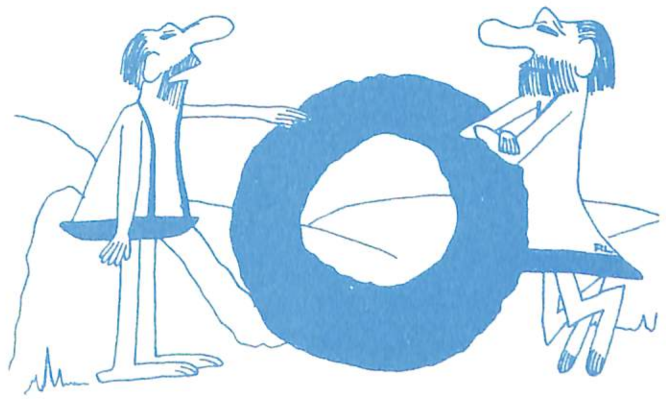
For those with even greater ambition, an oversize 9x9 math word array is known (on preliminary count) to "scramble" well over 400 admissible words of 3 to 9 letters:

```
10. N U M E R A T O R
    C O R O L L A R Y
    A S Y M P T O T E
    L O G A R I T H M
    T E S S E R A C T
    C O M P O S I T E
    I M A G I N A R Y
    P O S T U L A T E
    R E M A I N D E R .
```

Watch, here, for "included," therefore inadmissible, words. There are more than 30 of these, with TESSERACT itself incorporating at least 8 (CARESS, CARES, ARES--from are, metric unit of area measure--CARS, ACT, ARE, CAR, ERA). Proper nouns "entwined" in this 9x9 include a city in India and an ancient Scottish people. Reports of total word counts and point scores (on 20 longest words) will be welcomed.

The real "scrambled word" challenge, we remind you, is to come up with selections and orderings of mathematical terms ("math words") that will permit even higher "scrambled word" and point totals. In the case of our 9x9, nine "math words" have been selected, somewhat arbitrarily, from surely scores of possible mathematical terms, including plurals. The 9 then could be ordered among themselves (in the perspective of "scrambled word" searching) in 9!/2 or 181 440 essentially different ways. Is our 9x9 selection and ordering the best possible? Conceivably, but the odds against it, by chance, are countless billions to 1! Let's hear from you with your mathematical word grids and associated admissible word finds.

H.D.A.



WELL, I CAN SEE ITS SIGNIFICANCE AS AN ADDITIVE IDENTITY IN THE REAL NUMBER SYSTEM, BUT QUITE FRANKLY I DON'T THINK IT WILL EVER HAVE ANY PRACTICAL APPLICATION!

Paul Lochner in Mathematical Digest.

YOU'D BETTER NOT BELIEVE IT!

$\log_5 6 = e$?

Reporting, and indeed extending, a startling log-related result recently published in Mathematical Digest, University of Cape Town journal for high schoolers:

$$\text{If } \log_5 2 = a$$
$$\text{and } \log_5 3 = b,$$

determine $\log_5 6$. Now logarithms, being exponents, adding when the numbers of which they are exponents are multiplied, the result $\log_5 6 = a + b$ has certain merit ... but the following line of thinking evidences creative imagination. Clearly, in the assigning of symbols (above), a pattern has been present, and might logically be extended ...

$$\log_5 4 = c,$$
$$\log_5 5 = d;$$
$$\text{whence } \log_5 6 = e,$$

Mathematical Digest's tongue-in-cheek "result."

However, logically and consistently extending this nonsense somewhat further

$$\log_5 9 = i,$$

from which, with complete rigor, $\log_5 3 = \log_5 \sqrt{9} = i/2$. ... April Fool, everyone!

H.D.A.

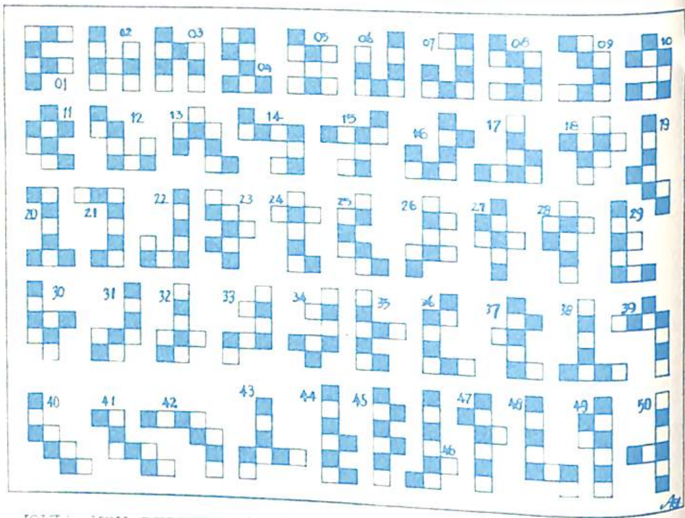
dia Log ue
with the editor

Logmaster Tom Bress (Riverdale chapter, Fort Myers, FL), having maintained his 1.000 batting average by demolishing every Log cryptogram in sight, sends the following as a challenge to Log readers. The verse is his own--and, forgiving a few anachronisms, possesses undeniable math-related charm.

ABRN KFRQYCNTNE TYT LOE,
JKMN JHA AI JQN FKTYY,
YB JQN RYFRXCINFNBRN JQNE ZA,
KBT HQKJ TA EAX MBAH,
YJ'L JQFNN OAYBJ ABN IAXF ABN IYDN BYBN!

Tom recently had the good fortune to locate a reprint copy of Sam Loyd's Cyclopedia of 5000 Puzzles, Tricks, and Conundrums, the posthumous compilation of much of "the best" of America's leading puzzlist.

Born in Elizabeth, NJ in 1841, Loyd became (for decades) a household word through his syndicated cartoons, problems, and math-related contests. Loyd is perhaps best known for his "15" sliding-block puzzle, the 19th century counterpart (at least in populaity) of Rubik's Cube. Delving into his Cyclopedia, Tom chanced upon one of Loyd's combinatoric classics: recalling the Cheshire Cat of "Alice" fame, the puzzle asks, "Was it a cat I saw?"



WOULD YOU BELIEVE [illegible].62 seconds by an energetic Logmaster who got himself a minicomputer for Christmas? You wouldn't! You're catching on fast! No, the fifty "octominoes" (above) have yet to be reassembled into their 10x10 checkerboard--the only major challenge published in a recent Log which no one yet has met.

The palidromic question was posed by Loyd in the form shown at left.

W
WAW
WASAW
WASISAW
WASITISAW
WASITAITSAW
WASITACATISAW
WASITAТISAW
WASITISAW
WASISAW
WASAW
WAW
W

Loyd's question, and Tom's question to Log readers: "In how many ways can 'Was it a cat I saw' be read in the letters of the given array? Let the Log know your total, or write Tom directly at 417 Canyon Dr. S., Lehigh, FL 33936.

In other correspondence, Logmaster Debbie Patonai, Mu Alpha Theta sponsor at St. Vincent-St. Mary High School, Akron, OH, writes that her classes devoured The Log's "scrambled searches" on the schoolday before Christmas, and asks for another set for pre-Easter use. We enclose them with pleasure (page 3). And with them relate their story. Unless you're better at it than we are, such "searches" are tough to produce and time consuming (but fun) to exhaust. We found ouselves in January with a medical problem and time on our hands. Hence the new flock of tried and proven searches. Not too profound a pursuit, we acknowledge--but it sure beats counting the drops in an intravenous or watching heartbeats chase each other across a screen!

In still other correspondence, Randy Secrist, a teacher in Fayetteville, NC, relates how he solved the graphic cryptogram "Classic Beginning" (Fall 1978 Log) ... on a microcomputer.

Can Rubik's Cubitis be cured? Mu Alpha Theta Governor Fred Hansen describes how he brought a conic section model to Geometry class, only to have a student pick it up, turn it every-which-way, and want to know how to "do" it!

CAROLINA CONTESTS ... from page 2

Copies of the Western Carolina University Mathematics Contest Book may be purchased for $[illegible], postpaid, from: Tuscola Mu Alpha Theta, Tuscola High School, Rt. 4, Box 350, Waynesville, NC 28786.

Any profits are earmarked for travel by math teams to competitions and students to Mu Alpha Theta in St. Louis, organizers note.

Convention, Back-to-School Issue

The Mathematical Log

VOLUME 27, NUMBER 1 OCTOBER 1982

Al-Biruni, Regular Polygons, & Flowers

by Ali R. Amir-Moéz
Texas Tech University

"Nature avoids whatever does not have a geometric solution," writes Abul-Raihan Al-Biruni (A.D. 973-1048) in his work, <u>Constructions of Chords in Circles</u> (ESTEKHRAJ-AL-OWTAR FI-AL-DAWAYER),

· استخراج الاوتار فی الدوایر

A "geometric" solution means a ruler-compass construction.

Even though Biruni suggests that there is no flower with <u>seven</u> or <u>nine</u> petals, he obtains a third-degree equation for construction of a regular <u>nonagon</u> in a unit circle. Before discussing this problem we shall study some regular polygons and flowers related to them.

1. <u>Preliminaries.</u>

In what follows s_n will denote the length of the side of the regular polygon of n sides, inscribed in a unit circle. We shall not give details for well-known facts.

Now we state without proof an important Theorem: A geometric configuration has a geometric construction if and only if parts of it can be obtained through linear or quadratic equations.

2. <u>Regular Polygons and Flowers.</u>

The side of an equilateral triangle inscribed in a unit circle, i.e., s_3, satisfies the equation

$$x^2 - 3 = 0.$$

The positive root is considered (Fig. 1). One may

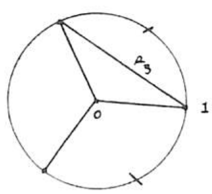
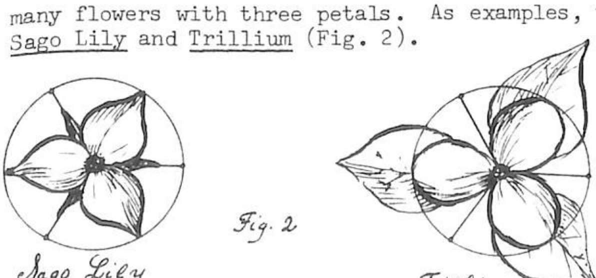


Fig 1

find many flowers with three petals. As examples, we give <u>Sago Lily</u> and <u>Trillium</u> (Fig. 2).



Sago Lily Fig. 2 Trillium

We observe that s_4 (Fig. 3) satisfies

$$x^2 - 2 = 0.$$

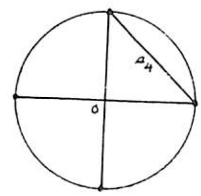
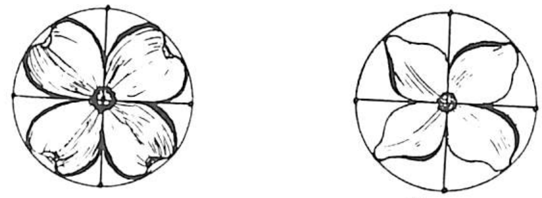


Fig. 3

<u>Dogwood</u> and <u>Mock Orange</u> are examples of flowers related to s_4 (Fig. 4).



Dogwood Fig. 4 Mock Orange

The side of the regular pentagon, i.e., s_5, satisfies

$$x^4 - 5x^2 + 5 = 0.$$

It is clear that this equation is quadratic in x^2.

SECRET OF SUCCESS

(Suggested by Tim Grose)

ABB CDE FGGH IF BIJG IK ILFDMAFNG AFH NDFJIHGFNG, AFH OPGF KQNNGKK IK KEMG.

--RAMS OTAIF

The roots are $x^2 = \frac{5 \pm \sqrt{5}}{2}$. Since $\frac{5+\sqrt{5}}{2}$ is larger than 1, it follows that

$$s_5 = \sqrt{\frac{5-\sqrt{5}}{2}}$$

which can be constructed geometrically (Fig. 5).

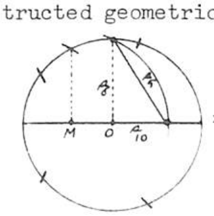


Fig. 5

(Continued on page 6)

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Editorially Logged

Another Great Year

From Los Angeles to St. Louis, at state level, and at well over one thousand school and junior college locations, this has been a great year for Mu Alpha Theta ... with highlights at least being reflected in the pages of The Mathematical Log. Welcome or welcome back to Mu Alpha Theta and to your Log--the first 8-page issue in Log history.

This school year, as last, we anticipate four Logs, to be issue-dated October, December, February, and April. Let's be hearing from you ... happenings and insights at state and chapter level, and individual interests, reactions, problems, and solutions. Deadlines for each issue are the 1st of the month of the previous issue--for example, December 1 for the February number (and deadlines, necessarily, are "for real").

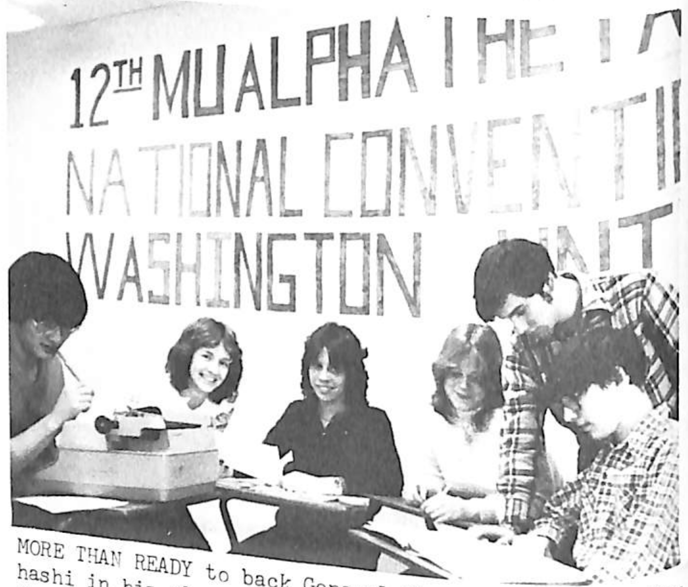
Election results (at national level) were not available in time to be reflected in this issue's masthead, but we're pleased with the healthy interest in these voluntary leadership positions. We note that Katherine Layton, Mu Alpha Theta past-president, will be our Mathematical Association of America representative. Kathy succeeds Dr. Robert Wilson, whose term is completed.

The Log welcomes new Logmasters Scott Berkenblit, N. Valley Stream, NY; Jim Cates, Ft. Deposit, AL; Joe Cates, Greenville, AL; Spencer Greene, Spring, TX; Tim Grose, Spring, TX; and Erik Lou, Houston, TX. A "Logmaster" patch also goes to Van Buren High School chapter, Van Buren, AR, for an outstanding 1140 total on the Geometry "Crosswords Challenge" (February Log). More about this in a future issue.

Mu Alpha Theta's challenge to chapters and individual members for the months ahead: blend mathematics and good fellowship, at chapter level and throughout your school community ... and communicate, through your Log. Share highlights and more ... of what we know will be another great year.

H.D.A.

Planning a Welcome



MORE THAN READY to back General Chairman Akehiko Takahashi in his plans for an outstanding 12th National Convention in St. Louis is this team of student enthusiasts from Wentzville High School chapter.

Old Logs need never die! Provide your school and community libraries with reference copies of each Log issue you receive.

EXOTIC NUMERALS

For those with a taste for the mathematically exotic, Log pagination this issue appears in Western, Thai-Lao, Korean, Javanese, and Arabic-Turkish digits.

Chapters Will Share Insights

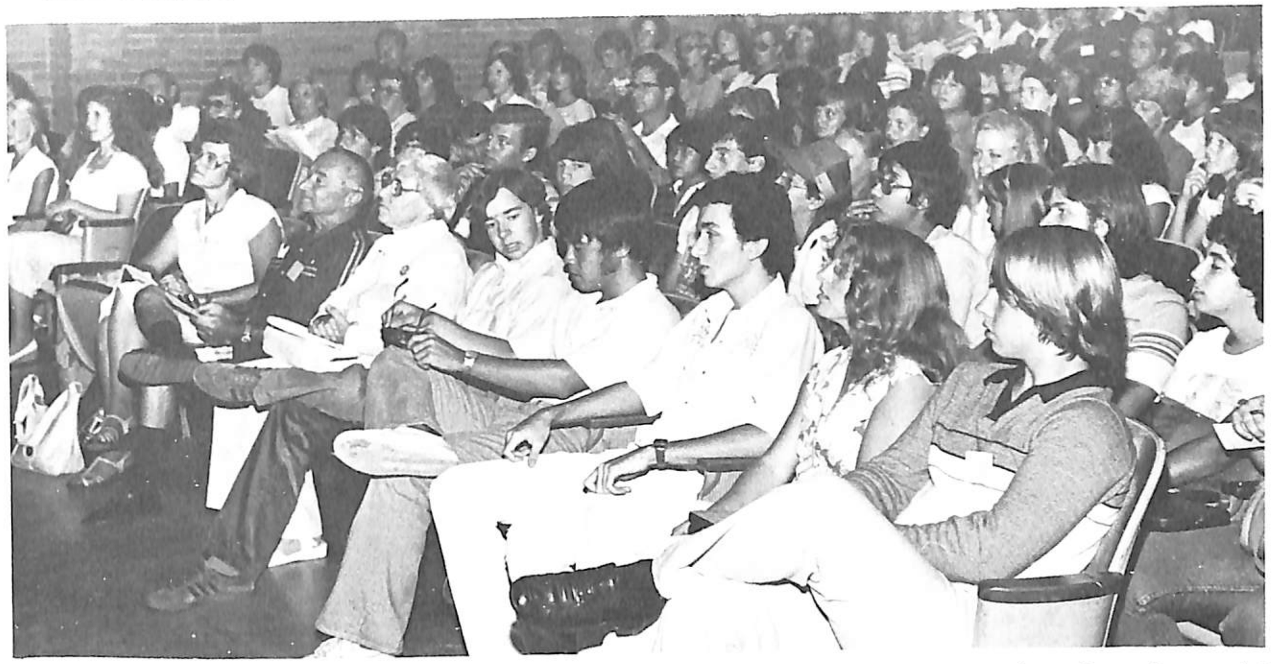
Mu Alpha Theta enthusiasts in three parts of the country--Florida, North Carolina, and Texas--have reported highly successful Mathematics Days and indicated willingness to share with other chapters their ideas.

Southwest Miami High School chapter (8855 S.W. 50th Terrace, Miami, FL 33165) featured at its Mathcomp '82 team and written competitions for five math levels; Rubik's Cube, calculator, computer, and "engineering" competitions, a "mathematics spelling bee," math trivia competition, and "mathematical scavenger hunt" ("a mad search, using mathematical clues"). Some 700 participated, chapter president Jim Knickerbocker reports.

Tuscola Senior High School chapter (Route 4, Box 350, Waynesville, NC 28786) considers its 3rd Annual Mathematics Field Day to have been a particular success. The chapter sponsor is John A. Goebel.

An all-day Math Tournament in West Orange, TX won outstanding press coverage, as evidenced by clippings submitted by Dr. Carol McGill. Dr. McGill may be contacted at 4405 Rue Des Fleurs, Orange, TX 77630.

Unique Personality of Mu Alpha Theta Reflected in Convention Scenes



WALL TO WALL PEOPLE at a General Session of Mu Alpha Theta's Los Angeles Convention show Convention as a time to meet new friends with similar interests from many parts of the nation. For much of each day the group breaks up, attending individual sessions on a great diversity of mathematics-related topics, then comparing notes at meals and in dorms. At General Sessions, though, sheer numbers add emphasis to the interest and enthusiasm that make a Mu Alpha Theta national gettogether a wholly memorable experience. Outstanding speakers are a highlight of annual Conventions.

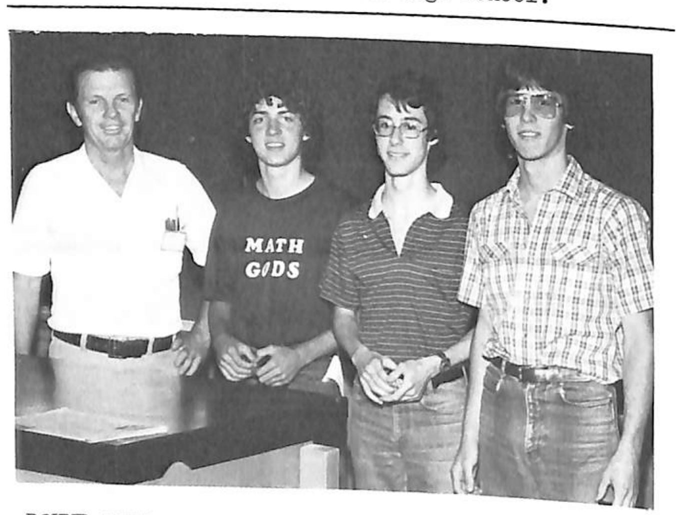
A CONVENTION HIGHLIGHT, presentation of the Robert Kalin Award to Mu Alpha Theta's outstanding student member. Edward Rimland (left), 1981 winner, is shown receiving the award check from Dr. Eugene P. Smith, Mu Alpha Theta national president. With an outstanding academic and extracurricular record, Rimland served as Mu Alpha Theta chapter president at Miami Coral Park Senior High School, Miami, FL. Dr. Kalin, donor of the award, is Professor of Mathematics Education at Florida State University and a former president of Mu Alpha Theta. The 1982 Kalin Award winner is to be determined at the St. Louis Convention and will be named at Convention and in the December Log.

GRISSOM HIGH SCHOOL claims its prize, the first award in the 1981 Convention Mathematics Competition, here being presented to a team representative by Fred Hansen. For most who attend, the "Math Competition," with its entries from across the country, is a major feature of a Mu Alpha Theta convention.

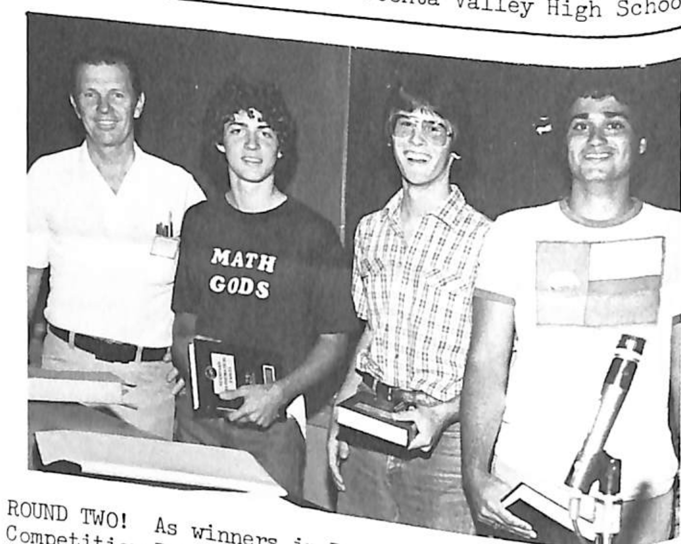
Convention Scenes

U.C.L.A. CONFERENCE ORGANIZERS I: Fred Hansen and his team of ten from La Canada High School.

U.C.L.A. CONFERENCE ORGANIZERS II: Ron Klint and six student enthusiasts from Crescenta Valley High School.



ROUND ONE! Los Angeles convention chairman Fred C. Hansen poses with the "top three" in Round One of the Mathematics Competition--David Rollins of Grissom High School, Nicholas Shapiro of New Trier High School, and Neal Fowler of Seholm High School. Round One winners were presented with Rubik's Cube key chains.



ROUND TWO! As winners in Round Two of the Mathematics Competition David Rollins, Neal Fowler, and Jerome De La Cruz (Marshall High School) received Mathematics Handbooks from Convention Chairman Fred C. Hansen.

Wide Range of Topics

Something for everyone (always good program planning!) has been the secret of success at Section Meetings at Mu Alpha Theta national gettogethers. Mathematicians from universities and industry, Mu Alpha Theta executive and sponsors, and student presenters share a surprising diversity of math-related ideas as these notes from U.C.L.A. presentations will attest:

Random Walks and the Gambler's Ruin: some of the simplest, yet most important, probabilistic models can be described within the context of gambling.

What Is Geometry? Did Euclid do it all? Is there reality after the Parallel Postulate?

Modern Aircraft Design Analysis: an overall view of applications of Matrix Algebra to obtain design data.

Rubik's Cube: Racing Back to Cube 1: gaining control of Rubik's Cube involves algebra in action, geometric visualization in 3-D, and the design of kinesthetic algorithms (see photo).

How Queing Theory Can Save the World: an introduction to the theory and history of queing, with real world applications.

The Cantor Set: a set with zero length, yet as many points as the real line--weird!

Stretching Euler's Formula: applying a formula for convex polygons to topology.

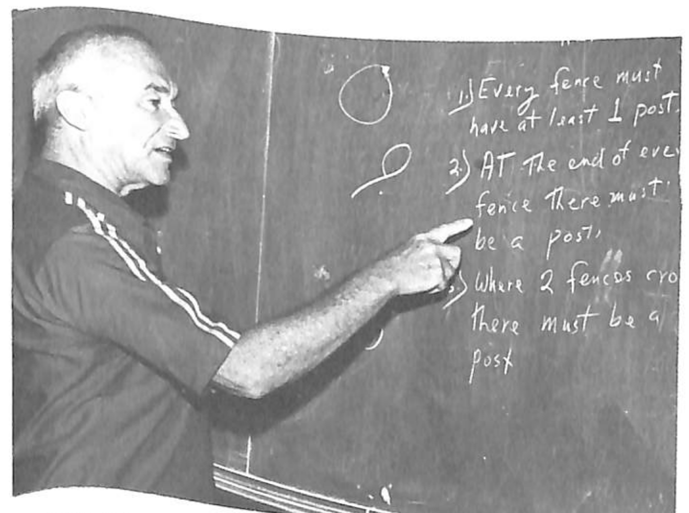
Math-Science and Crime: a criminologist with the Los Angeles Police Department discusses the role of mathematics in police work.

Rewarding Careers from Math-Science: a discussion of high school and college training needed for many different careers.

Such a listing as this being obvious "grist for the mill" for chapter program planners, similar highlights from St. Louis sessions will be offered in a future Log.

Convention Scenes

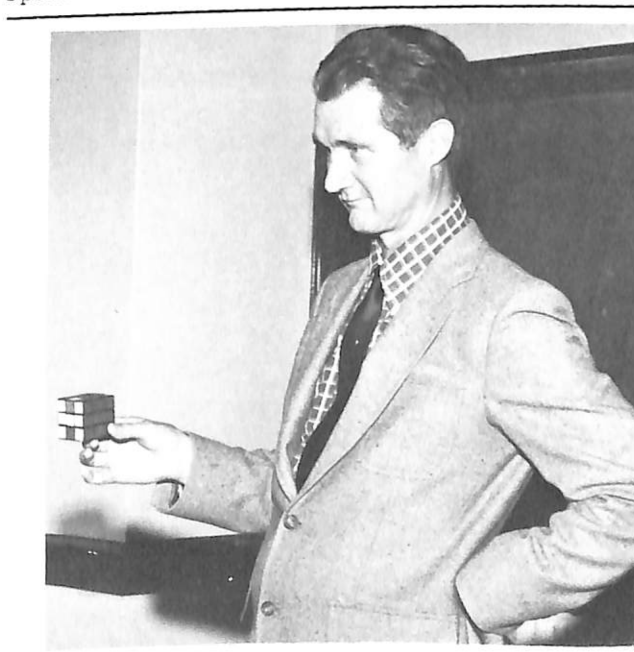
YOU AND THE MOB! Fingerprint classification, an unusual and fascinating mathematical application, was presented by students from the Milwaukee Trade Tech Math Club, Milwaukee, WI. Here, fingerprint characteristics are being pointed out in a workshop session.



EULER'S FORMULA DEVELOPED. Harry D. Ruderman, mathematics department chairman at Hunter College High School, New York, NY--and Mu Alpha Theta--and Mathematical Buds coordinator for Mu Alpha Theta--presents a flawed proof of the 1:1 correspondence between points on [0,1] and a unit square--and challenges his audience to find the flaws. Each presentation was given on several days.

1981 UCLA Convention photos by Frank Moody

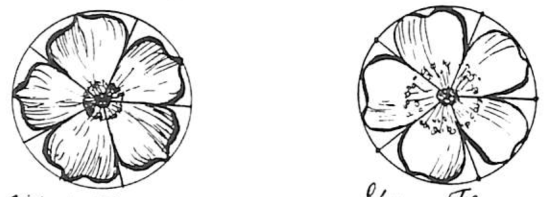
CHARACTERISTIC POSE? No, a chance to stretch between addresses, as Dr. George Fischbeck prepares to tell Mu Alpha Theta that "You Gotta Fail to be a Success." A meteorologist and popular Los Angeles television "weatherman," Doctor Fischbeck urged members not to limit their ambitions by fear of failure. Mu Alpha Theta has been fortunate in attracting outstanding speakers for General Sessions at its Conventions.



CUBE CONTROL? In one of the most popular sessions at Mu Alpha Theta's UCLA Convention, Professor Herbert Taylor, University of Southern California, presents "gaining control" of a Rubik's Cube as involving "algebra in action, geometric visualization in 3D, and the design of kinesthetic algorithms." Cubes sprang up like mushrooms in dormitory rooms in anticipation of, and following, Professor Taylor's presentations.

Al-Biruni ... FROM PAGE 1

Wild Rose and Globe Flower are two examples of five-petal flowers (Fig. 6).



Wild Rose Fig. 6 Globe Flower

The simplest of all regular polygons is the regular hexagon (Fig. 7). One notes that s_6 satisfies
$x = 1$.

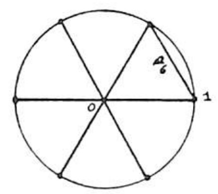
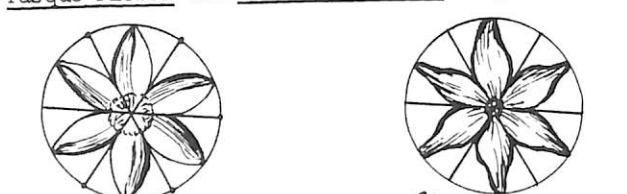


Fig. 7

As examples of six-petal flowers one can mention Pasque Flower and Yellow Jessamine (Fig. 8).



Pasque Flower Fig. 8 Yellow Jessamine

The golden section polygon is the regular decagon (Fig. 9).

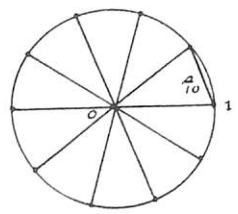
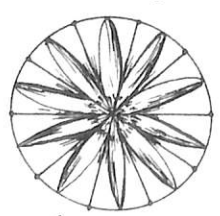


Fig. 9

The side of this polygon, i.e., s_{10}, satisfies

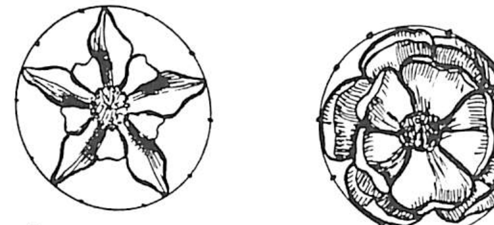
$$x^2 + x - 1 = 0$$

which is the equation of the golden section. Stickweed is an example of a ten-petal flower (Fig. 10).



Stickweed
Flower of The Golden Section
Fig 10

The plant, with its flowers of light golden color, stands out like a constellation on a clear night. There are other flowers of ten petals, such as Columbine and Magnolia (Fig. 11). These flowers actually have two sets of five petals.



Columbine Fig. 11 Magnolia

3. Regular Nonagon.

Dividing a circle into nine equal arcs, we observe that each arc corresponds to 40° (Fig. 12). If

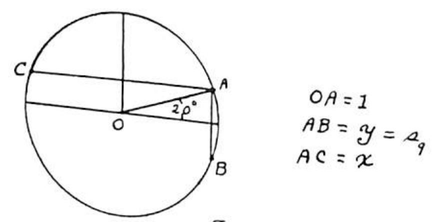


Fig. 12

AB = y is the side of the regular nonagon in a unit circle, then

$$\frac{y}{2} = \sin 20^\circ.$$

The identity

$$\sin 3t = 3 \sin t - 4 \sin^3 t$$

applied to $3(20^\circ) = 60^\circ$ yields

$$\sin 3(20^\circ) = 3 \sin 20^\circ - 4 \sin^3 20^\circ.$$

Thus,

$$\frac{\sqrt{3}}{2} = \frac{3y}{2} - 4\frac{y^3}{8}$$

which implies that s_9 satisfies

$$y^3 - 3y + \sqrt{3} = 0.$$

This equation, having $\sqrt{3}$ in it, did not please Al-Biruni. So he chose AC = x and obtained

(1) $$x^3 - 3x = 1.$$

He then determined an accurate approximation for x, i.e.,

$$x = 1.8793852418.$$

(In order to obtain (1) one may use

$$\frac{1}{2} = \cos 60^\circ = 4 \cos^3 20^\circ - 3 \cos 20^\circ.)$$

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diaLogue
with the editor

The spirit of "diaLogue" was admirably captured by Convention photographer Frank Moody in this candid shot of a Los Angeles mathematical recreations session. Accordingly, while Editors who feature their own picture are (or perhaps should be) suspect, we're using it to head this issue's column in the hope of aiding Editor-spotting and encouraging just such dialogue, both at St. Louis and (through correspondence) over the following year! Your "input" means a better Log. Let's hear from you.

* * *

Taking in the full breadth and richness of mathematical thought--at whatever level--calls for an uncommon perspective. Something of such a perspective is, we submit, provided in two remarkable anthologies that a Log reader has very thoughtfully brought to our attention. The references are Fantasia Mathematica (Simon and Schuster, 1958) and The Mathematical Magpie (same publisher, 1962), compilations of prose, poetry, quotes, cartoons, and much more, by the versatile American anthologist, Clifton Fadiman. What a weekend of mathematics related reading! Fantasia includes a memorable trio of topological tales, "No-Sided Professor" and "The Island of Five Colors," by Martin Gardner (better known for his expository prose than for his fiction), and "A Subway Named Moebius," by A.J. Deutsch. Magpie reprints the Stephen Leacock classic, "A, B, and C--The Human Element in Mathematics," and Arthur C. Clarke's science fiction gem from the earliest years of the computer age, "The Nine Billion Names of God." Speaking of "Nine Billion Names," written in May, 1952, Clarke reads this unforgettable tale on Transit of Earth (Caedmon stereo, TC 1566) ... not surprisingly, it makes for good listening as well as good reading.

* * *

Logmaster Tom Bress of Fort Myers, FL, posed the question in our April "diaLogue": In how many ways can the palindrome, "Was it a cat I saw" be read in the letters of the pattern at left? The question recalls the Cheshire Cat of "Alice" fame, and is due to American master-puzzlist Sam Loyd (1841-1911). From Logmaster Betty Lichtenberg, Tom has received the following answer--and generalization: Sam Loyd's 13-letter palindrome can be "read" in

```
      W
     WAW
    WASAW
   WASISAW
  WASITISAW
 WASITATISAW
WASITACATISAW
 WASITATISAW
  WASITISAW
   WASISAW
    WASAW
     WAW
      W
```

$$[4(2^6 - 1)]^2, \text{ or } \underline{63\ 504}$$

ways. Generalizing, for a palindrome of length 2n + 1 letters, the number of ways is given by

$$[4(2^n - 1)]^2.$$

* * *

"Prove that the sum of 2 different squares, multiplied by the sum of 2 different squares, gives the sum of 2 squares."

That question (its general solution, not a mere confirming illustration) calls for a fair page of algebra--and therein rests a story. For on the night of December 3, 1881, a remarkable Englishman lay awake in bed and fully solved that problem ... in his head.

The Englishman was The Reverend Charles L. Dodgson, M.A., better known to the young and the not-so-young by his pen-name, Lewis Carroll. The problem and the "in the head" solution the creator of "Alice" set down the next morning, as had become his custom. Ultimately he shared it, in a surely unique collection of 72 such "mental" examples, his Pillow-Problems Thought Out During Wakeful Hours ("having been solved, in the head, while laying awake at night")--see, for a modern edition, Mathematical Recreations of Lewis Carroll: Pillow Problems and A Tangled Tale (New York: Dover Publications, Inc., 1958).

Other examples of "pillow-problems" to which Dodgson records "in the head" solutions:

"Given the 3 altitudes of a Triangle: construct it."

"If a, b be two numbers prime to each other, a value may be found for n which will make $(a^n - 1)$ a multiple of b."

"There are 3 bags, each containing 6 counters; one contains 5 white and one black; another, 4 white and 2 black; the third, 3 white and 3 black. From two of the bags (it is not known which) 2 counters are drawn, and prove to be black and white. What is the chance of drawing a white counter from the remaining bag?"

Remember, "in the head"!

TRANSFINITE PERSPECTIVE

C XTK T BWWPCHZ MHSW TLMDR GTRXWGTR-CSN--RXTR C NTI CR TPP. KWYRX LWUMHK KWYRX ITN OWJWTPWK RM GW--RXW LUNN THK RXW ROTHN-TLUNN. C NTI--TN MHW GCZXR NWW RXW PMOK GTUMO'N NXMI--CR MB JWHDN MO WJWH RXW PMOK GTUMO'N NXMI-- CR MB JWHDN MO WJWH RXW

--ICHNRMH SXDOSXCPP

"Reaching 100," a number game that has been around for some time, puts a premium on certain elementary insights, and (along with its simple variants) can be fun to master and perhaps further develop.

As usually played, "Reaching 100" involves two participants, the first picking and announcing a number between 0 and (arbitrarily) 10. The second player adds to this figure a number between (equally arbitrarily) 1 and 10. Players continue to alternate, adding to existing totals numbers between (in this version) 1 and 10. The first to reach a specified total, 100, wins.

Clearly, in this two-player standard version, whoever first reaches 89 can win (if his opponent adds x, he responds with 11 - x); further, whoever reaches 78 similarly can reach 89, and so on. A winning strategy evidently follows, but with an unsuspecting opponent the "1, 12, ..., 67, 78, 89" "controlling number" sequence need not, and probably should not, be entered until close to the end.

Interesting variants obviously await development and exploration. For example, how about reaching $1 using 1¢, 5¢, 10¢, and 25¢ coins; or $100 using (as in the Bahamas) $1, $3, $5, and $10 bills; or a stipulated amount of postage using certain available postage stamp denominations?

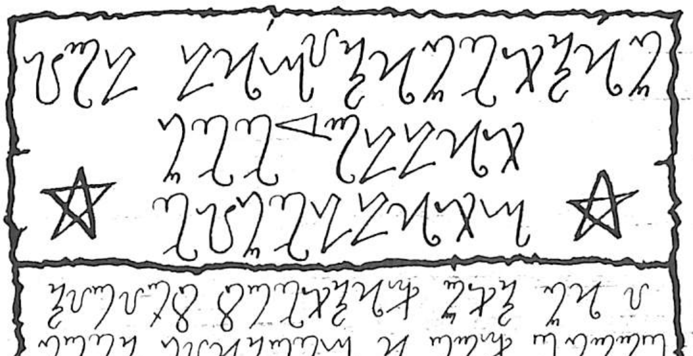
St. Louis Challenges!

Conventions "spin off" challenges! The Editor has made it a practice to come to his Mu Alpha Theta talks armed with new puzzlers created for the occasion, then to offer such challenges wider circulation in a subsequent Log.

Since such competitions have no immediate deadlines, those attending have no unfair advantage--though it is fun to "take on" such a challenge in a Convention setting with new-found friends. Solutions are published in The Log in due course.

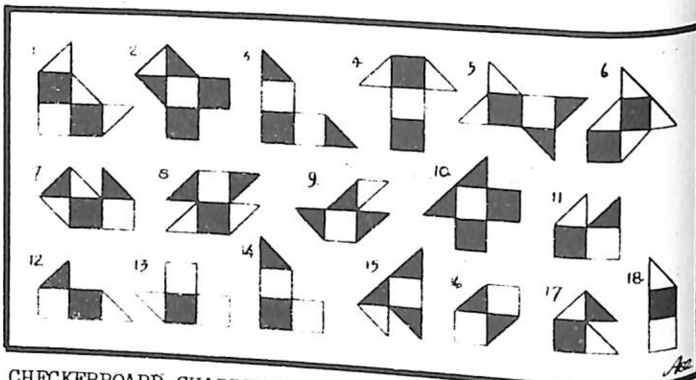
On this page are three of this year's challenges. Give them a try, and let's hear how you do.

FROM THE BEYOND



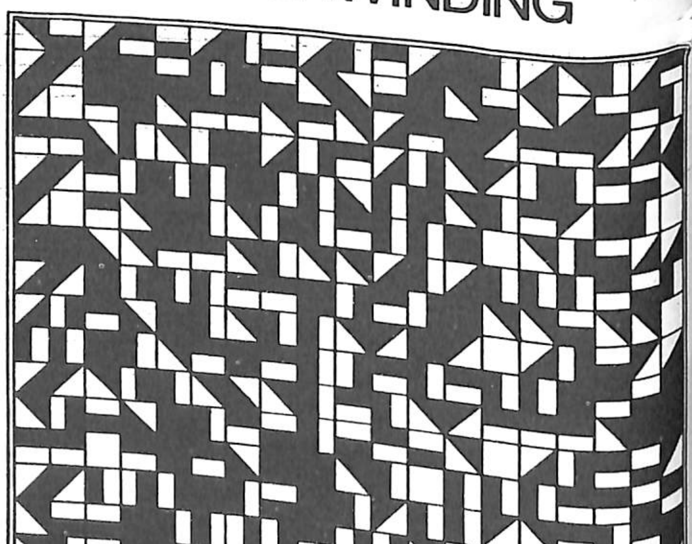
FROM THE BEYOND! This year's Mathematical Log Hallowe'en Challenge (as previewed at St. Louis) takes its cue from what probably is the most famous ghost scene in English Literature--and encyphers the ghostly utterances in a traditional alphabet in which witches kept their spells. Can you "decode" it? The key will be given next issue. For those with an interest in such unusual alphabets, compare with the Runic Alphabet in which last year's Hallowe'en Challenge was written (see Fall 1981 Log, pp. 4, 6).

FRACTURED CHECKERBOARD



CHECKERBOARD CHALLENGE with a difference! This latest "fractured checkerboard" (a St. Louis special) breaks "squares" on the diagonal, adding additional challenge to the kind of puzzle that has been popular in earlier Logs. Can you reassemble the 18 pieces to form a perfect, 8x8 checkerboard, squares alternating dark and light? A "Logmaster" award awaits the first to do so!

PATTERN UNWINDING



THE TOUGHER THE PROBLEM, THE HARDER THEY TRY has been our discovery before Mu Alpha Theta audiences, hence "Pattern Unwinding," mathematical "art" produced as an open-ended St. Louis challenge. Can you discover a pattern? Having identified the pattern can you extend it to complete the final row? Note that the overall pattern can be regarded as composed of unit squares, each divided and/or shaded (or unshaded) in one of ten ways ... hence the likelihood of a numerical sequence.

MU ALPHA THETA INITIATION CEREMONY

PRESIDENT:

Candidates, you have presented yourselves for initiation in Mu Alpha Theta, an international honorary mathematics society. It is an honor to you to have been selected for membership in this society which has over 2000 chapters in all fifty states and in some foreign countries. You each meet the requirements which include work done with distinction in college mathematics. You have demonstrated that you possess the qualities of industry, initiative, and reliability.

VICE-PRESIDENT:

Before you is a replica of the society's insignia. Its blue represents truth as unlimited as the sky. The gold shines as a symbol that mathematics is a valuable treasure. The insignia represents, above all, a supreme law of mathematics…one combining the mystery, the challenge, and the beauty of numbers with an indispensable law of geometry.

SECRETARY:

Mu Alpha Theta was formed to engender keener interest in mathematics, to develop sound scholarship in the subject, and to promote enjoyment of mathematics among students.

PRESIDENT:

The society's name carries with it the meaning of the organization. **Mu** stands for the Greek word for learning; **Alpha** stands for the Greek word for truth; and **Theta** stands for the Greek word for service. I invite you now to stand and to repeat after me the pledge which admits you into full membership.

PLEDGE: "I, __________________, do solemnly promise to uphold the standards of Mu Alpha Theta and to remain true to its purpose, and I do solemnly pledge allegiance to my fellow members and I promise to aid them in the search for mathematical truths."

SPONSOR: I now declare you members of Mu Alpha Theta. It is my pleasure to welcome you into our organization and to commend you on your accomplishments in mathematics. I challenge you to always strive to do your best while you are here at ECCC and to dedicate yourself to cultivating a well-rounded life, a life of service to others, and a life of honor in your community.